Modeling the Throughput of 1-persistent CSMA in Underwater Networks

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***Abstract*—The aim of this paper is to present a model for the throughput of the 1-persistent CSMA protocol in underwater networks, where the typically large propagation delay with respect to the packet transmission time requires to take into account the spatial distribution of the nodes. Our model is developed based on the analysis carried out in [1] for the non-persistent CSMA protocol. Our results show that the 1-persistent CSMA model developed by Tobagi and Kleinrock is still valid as an approximation, with a few small adjustments, even though it considers an equal propagation delay for all pairs of nodes in the network. The proposed model is validated against simulation results based on the network simulator OMNeT++.**

***Index Terms*—Underwater acoustic networks, CSMA, Medium Access Control, throughput analysis, simulation.**

I. Introduction

Carrier Sense Multiple Access (CSMA) is a Medium Access Control (MAC) protocol based on random access with channel sensing prior to each transmission. In underwater networks, CSMA has been playing a significant role mainly for two reasons: i) it is simple to implement in real hardware for field experiments, as it does not require additional capabilities such as network synchronization; ii) even when the sensing range is equal to the reception range (as is typically the case for most off-the-shelf commercial modems to date), CSMA still helps avoid some collision events, such as those caused by a node starting to transmit when it is overhearing a packet not meant for itself. Indeed, CSMA has been among the first protocols to be tested in simple MAC experiments in small networks [2]. In its collision avoidance flavor (CSMA/CA), which makes use of a preliminary 4-way handshaking, CSMA has been investigated, analyzed and simulated in different scenarios and under different assumptions, including hybrid approaches [3]. For example, the authors in [4] consider TDMA features to enhance the performance of a CSMA-based access protocol by implementing a network synchronization protocol and by having all nodes estimate the propagation delay towards all neighbors in order to predict collision events; [5] employs CSMA/CA as a MAC scheme on top of Orthogonal Frequency-Division Multiplexing (OFDM) for multiple swarming autonomous underwater vehicles; a MACA-based protocol is proposed in [6] and analyzed by taking into account the significant propagation delay of underwater acoustic channels (compared to control and data packet transmission times) and the typically severe data losses experienced under water; a virtual full-duplex scheme exploiting the underwater acoustic propagation delay is proposed in [7] to improve the performance of CSMA/CA.

In the following, we will focus on the "sense-before-transmit" version of CSMA, i.e., without collision avoidance mechanisms based on preliminary handshakes. According to the terminology established by Tobagi and Kleinrock [8] (TK for short), in non-persistent CSMA (np-CSMA) the node backs off and reschedules a later transmission attempt whenever the channel is sensed busy.

In a more general version, named p-persistent CSMA (p-CSMA), time is slotted and a node senses the channel in each slot: if the channel is sensed idle, the node transmits with probability p, and refrains with probability $1-p$; if the channel is busy, the node backs off and performs sensing again in the next slot.

A specific version of p-persistent CSMA sets $p = 1$ and is named 1-persistent CSMA (1p-CSMA). The latter technique may potentially achieve better channel utilization than np-CSMA, but if two users sense the channel to be busy within a short time window, it becomes highly likely that their transmissions will collide [8].

Theoretically, CSMA makes it possible to achieve maximum channel utilization S, defined as the ratio of the amount of time spent for correct transmissions over the total time. However, underwater networks typically exhibit large propagation delays (denoted here as τ), where "large" means that τ is a non-negligible fraction of, or may even be larger than, the packet transmission time T_D. The performance of CSMA is studied by Tobagi and Kleinrock in [8] assuming that τ is constant across all node pairs, which is appropriate only when $\tau \ll T_D$. Whenever this condition is not verified, it becomes inaccurate to approximate τ as equal between all pairs of nodes, and the spatial distribution of the nodes must be explicitly taken into account. In fact, a node may achieve a successful transmission if any concurrent transmitters are located sufficiently far away, so that their related receptions do not overlap in time. In the np-CSMA case, an extension of the TK analysis has been presented by Koseoglu and Karasan [1] who studied np-CSMA taking into account spatio-temporal effects, and obtained a model similar to the TK model in terms of equations, but that fits simulation results much better for significant propagation delay scenarios. In fact, the TK model tends to underestimate the throughput when τ is not negligible with respect to T_D.

In this paper, we extend the results in [1] by analyzing the throughput and success probability performance of 1p-CSMA in networks with large propagation delays. We show that the different traffic pattern with respect to np-CSMA leads to different time-varying packet arrival rates than shown in [1] for the np-CSMA case, and discuss an approximation to these rates. The throughput analysis based on these results is shown to provide better accuracy when predicting 1p-CSMA performance figures with respect to the TK model in [8]. As introduced at the beginning of this section, our analysis is quite relevant to underwater networking, as many protocol stacks are based on some form of CSMA, possibly even natively provided by the modem [9].

The rest of this paper is organized as follows. In the next section we briefly describe the network scenario and introduce the notation used. In Section III we investigate the rates introduced by Koseoglu and Karasan in their paper [1] and characterize their behavior for large values of a. This will be the basis for the development of our model as we revisit the TK model in Section IV based on the results in Section III. In Section V, the model is compared to simulation results obtained using the OMNeT++ network simulator. Finally, in Section VI, we will discuss the model and its possible future refinements before drawing our conclusions.

II. Network Scenario

For completeness, we now introduce the same terminology used in [8]. Let G denote the aggregate packet generation rate in the network. Let τ be the largest propagation delay of the network, and call T_D the packet transmission time. Let $a = \frac{\tau}{T_D}$ denote the ratio between the largest propagation delay τ over the packet transmission time T_D.

The scenario is the same as in [1] except that we assume a finite number N of users $\{n_0, n_1, \dots\}$ to be uniformly distributed inside a circle of diameter a with the receiving access point (AP) located at the center. As in [1], we assume that physical distances and "time distances," or propagation delays, are equivalent. From this point onward, all positions and distances considered in the model will be considered expressed in terms of time. Each node generates packets according to a Poisson process of parameter $\frac{G}{N}$. The described scenario is illustrated with reference to Fig. 1. Each node n_i of the network is located at the polar coordinates pair (r_i, θ_i) which is referenced to the center of the circle, where the AP is located. The distance between two nodes n_0 and n_1 as a function of their polar coordinates can be calculated as

$$d(n_0, n_1) = \sqrt{r_0^2 + r_1^2 - 2 r_0 r_1 \cos(\theta_0 - \theta_1)}. \quad (1)$$

We assume that the N users are uniformly distributed within the circle, i.e., that their polar coordinates have the following probability density functions (pdfs), for $0 \le i \le N-1$

$$f_R(r_i) = \frac{2 r_i}{a^2/4}, \quad 0 \le r_i \le \frac{a}{2} \quad (2)$$

$$f_\Theta(\theta_i) = \frac{1}{2\pi}, \quad 0 \le \theta_i \le 2\pi \ . \quad (3)$$

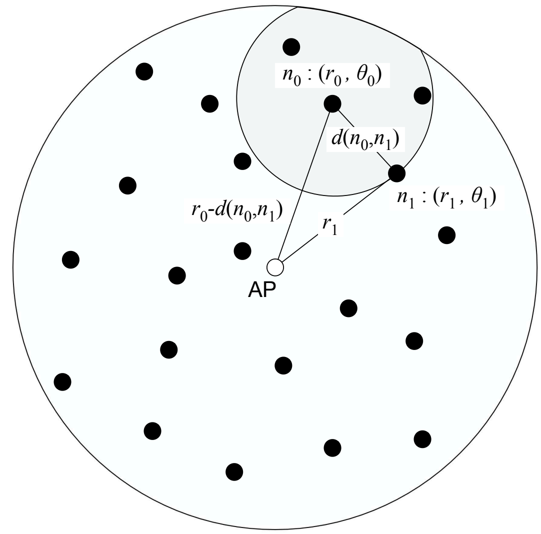

Figure 1. The network scenario. Nodes are uniformly distributed inside a circle whose diameter measures a time units, normalized to the packet transmission time T_D. (Adapted from [1].)

III. Characterization of packet rates during and after reception at the AP

Along the lines of [1], we introduce two rates, namely $\lambda^{\mathrm{st}}(t)$ and $\lambda^{\mathrm{end}}(t)$. The former represents the average rate of colliding packets from the beginning of the reception of a packet, transmitted by a node n_0, at the AP. The latter expresses the rate of incoming packets at the AP after the completion of the reception of a packet transmitted by a node n_0. In the following, we analyze both rates in details.

A. Average rate of colliding packets after packet reception: $\lambda^{\mathrm{st}}(t)$

The rate $\lambda^{\mathrm{st}}(t)$ represents the rate of colliding packets that would arrive at the AP when a packet sent by a node n_0 is received. We set $t = 0$ to be the epoch when the AP begins to receive the packet from node n_0. First of all, notice that, because of the finite duration of a packet transmission (1 time unit, normalized to the packet transmission time T_D), the rate $\lambda^{\mathrm{st}}(t)$ takes non-zero values only for $t \in [0, 1]$. Given a node n_1 located at (r_1, θ_1), this node can collide with the transmission of n_0 so far as $t < d(n_0, n_1) + r_1 - r_0$. Another collision could take place if $t > d(n_0, n_1) + 1 + r_1 - r_0$ but this is impossible as long as $t \in [0, 1]$. Taking into account the preceding assumptions, the calculation of the rate is very similar to the one in [1], except for the limited support of the function. This leads to the following linear approximation of $\lambda^{\mathrm{st}}(t)$:

$$\lambda^{\mathrm{st}}(t) = \begin{cases} G\dfrac{a-t}{a} & \text{if } t \le \min(1, a) \\ 0 & \text{if } t > \min(1, a) \end{cases} . \quad (4)$$

B. Average incoming packet rate after reception: $\lambda^{\mathrm{end}}(t)$

The rate $\lambda^{\mathrm{end}}(t)$ represents the rate of incoming packets arriving at the AP after the completion of the reception of a packet. Here we set $t = 0$ as the epoch when the AP completes the reception of a packet sent by n_0. Note that, unlike in the np-CSMA case, in 1p-CSMA a node might generate a packet while the channel is busy and withhold the transmission until the channel becomes free again.[1] In the following, we will refer to this type of node as a backlogged node. Three different cases are possible:

- According to the considerations in [1], if t_0 is the time when a node $n_0 \equiv (r_0, \theta_0)$ begins its transmission, a second node $n_1 \equiv (r_1, \theta_1)$ becomes aware that the channel is idle at time $t_0 + d(n_0, n_1) + 1$ (the normalized packet transmission time equals 1); the transmission by n_0 is fully received at the AP at time $t_0 + r_0 + 1$: therefore, a transmission by n_1 not due to backlogging (referred to as "normal" transmissions in the following) could arrive at the access point whenever $t \geq d(n_0, n_1) + r_1 - r_0$;
- Any node n_1 is aware of n_0's transmission for a time interval of normalized duration 1; node n_1 may become backlogged during this period: in this case, it will transmit right after n_1's transmission is over and therefore its backlogged packet will be received at the AP at time $t = d(n_0, n_1) + r_1 - r_0$;
- Another case must be taken into account, which does not occur if $a < 1$: there could be a node n_1, sufficiently far from the AP, which can start its transmission before being aware of the transmission of n_0 and whose packet will arrive at the AP after the complete reception of n_0's packet. This can happen as long as $t < d(n_0, n_1) + r_1 - r_0 - 1$.

Thus, we can compute the rate $\lambda^{\text{end}}(t)$ as the superposition of the rates at which the three cases above occur.

C. Considerations about the rates

We remark that an equality of the kind $t = d(n_0, n_1) + r_1 - r_0$ represents an ellipse. In fact, rewrite the former expression as $d(n_0, n_1) + r_1 = t + r_0$ and refer to Fig. 1. Now, consider a time t and, without loss of generality, a node $n_0 = (r_0, 0)$. The set of points (r_1, θ_1) that satisfy the relationship $d(n_0, n_1) + r_1 = t + r_0 = \text{constant}$ is an ellipse with the AP, located at $(0, 0)$, and node n_0 as foci.

It is useful to determine how much the area $A(t)$ of this ellipse, which is a function of r_0 and t, varies as t increases by an infinitesimal amount Δt. For simplicity, ignore the fact that the ellipse must be contained inside the circle with diameter a. From basic geometry, the area of an ellipse is

$$A(t) = \pi \alpha(t) \beta(t) \ , \tag{5}$$

where $\alpha(t)$ is the semi-major axis, $\beta(t)$ the semi-minor axis, and $t = d(n_0, n_1) + r_1 - r_0$ as above. Recall once again that all locations and distances are expressed in terms of time. We stress that the sum of the distances from the foci is always $2\alpha(t)$, and that the distance of each focus from the ellipse center is $f = \sqrt{\alpha(t)^2 - \beta(t)^2}$.

[1] Along the lines of [8], we assume that the nodes can hold at most one packet in their buffer.

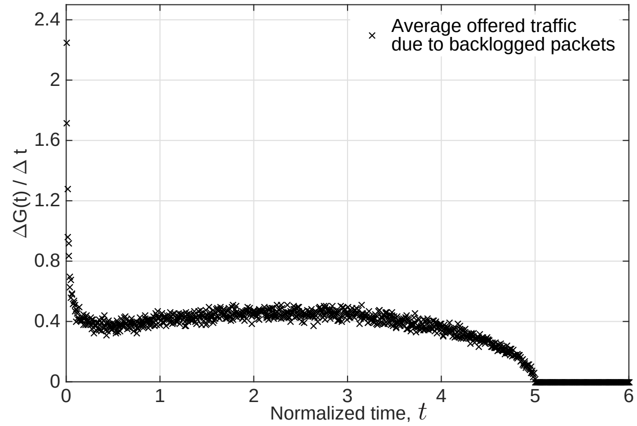

Figure 2. The average rate of incoming packets after full reception at AP due to backlogged packet for $a = 5$ and $G = 2$. Notice that the function integrates to G.

In our case, the distance between the foci is constant. Therefore, we can express the area as a function of t and r_0 via the following substitutions: $r_0 + t = 2\alpha(t)$ and $2f(t) = r_0$. These lead to

$$\alpha(t) = \frac{r_0 + t}{2}; \qquad \beta(t) = \sqrt{\left(\frac{r_0 + t}{2}\right)^2 - \left(\frac{r_0}{2}\right)^2} \ , \tag{6}$$

and the ellipse area as a function of r_0 and t can be found by substituting (6) into (5). Finally, for Δt sufficiently small, we can write

$$\Delta A(t) = \frac{\mathrm{d}A(t)}{\mathrm{d}t} \Delta t \ , \tag{7}$$

where $\mathrm{d}A(t)/\,\mathrm{d}t = \pi \left(r_0^2 + 4r_0 t + 2t^2\right) / 4\sqrt{t(2r_0 + t)}$. Notice that, for $t \to 0$ the function grows to infinity as $t^{-\frac{1}{2}}$ and is therefore integrable. Instead, for large values of t the function grows linearly. However, since the ellipse is contained in a circle of diameter a, $\Delta A(t)$ will be constantly 0 for $t > a$. Recalling that the users are uniformly distributed inside a circle of diameter a, as $N \to \infty$, an area $\Delta A(t)$ will offer a traffic

$$\Delta G(t) = \frac{4G}{\pi a^2} \Delta A(t) \ . \tag{8}$$

Because of the linear relation between $\Delta A(t)$ and $\Delta G(t)$ in (8), the rate due to backlogged users will assume the behavior described by Eq. (7), at least when $t \ll a$.

In Fig. 2, we show $\Delta G(t)/\Delta t$ vs. t for $a = 5$ and $G = 2$, by also considering the only portion of the ellipse contained inside the circle of diameter a around the AP, and by averaging over r_0. In the figure, we observe the asymptotic behavior for $t \to 0$ that has been discussed above, and also that the rate is exactly 0 for $t > a$. We note that $a = 5$ may be representative of an underwater network where nodes communicate 100-Byte packets at 1 kbps over a maximum network radius of 2 km (assuming the propagation delay to be constant and equal to 1.5 km/s as a first-order approximation).

With reference to $\lambda^{\text{end}}(t)$, the ellipse argument can also be exploited for the following consideration. The transmission

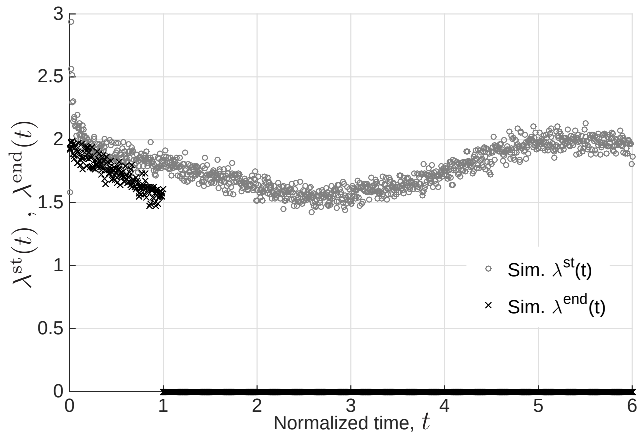

Figure 3. The simulation results for $\lambda^{\rm st}(t)$ and $\lambda^{\rm end}(t)$ for $a = 5$ and $G = 2$.

of newly generated packets is possible for $t > d(n_0, n_1) + r_1 - r_0$ and also for $t < d(n_0, n_1) + r_1 - r_0 - 1$. The first inequality describes the area inside the ellipse determined by the equation $t = d(n_0, n_1) + r_1 - r_0$, whereas the second inequality describes the area outside the ellipse determined by $t = d(n_0, n_1) + r_1 - r_0 - 1$. By calling this area $\mathcal{E}$, we have $4\mathcal{E}/(\pi a^2)G \to G$ for $a \to +\infty$.

Fig. 3 shows the simulated values of $\lambda^{\rm st}(t)$ and $\lambda^{\rm end}(t)$. From the plot of $\lambda^{\rm end}(t)$, we can observe the effect of the backlogged transmission for small values of t, and also the fact that up to $t = a$ the rate is quite close to G. In addition, we note that the simulation of $\lambda^{\rm st}(t)$ is very well approximated by the model in (4).

D. Asymptotic Behavior

We start by analyzing the rate $\lambda^{\rm st}(t)$. As a grows to infinity, it can be easily seen that $\lambda^{\rm st}(t)$ uniformly converges to the function:

$$\lambda^{\rm st}(t) \to \lambda^{\rm st}_\infty(t) = \begin{cases} G, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{for} \quad a \to \infty \ . \tag{9}$$

For the rate $\lambda^{\rm end}(t)$, we have the following considerations. First of all, from Eqs. (7) and (8), we notice that $\Delta G(t)$ is directly proportional to G; moreover, noting that the average value of r_0 is directly proportional to a (the average value of r_0 for a node uniformly distributed inside a circle is one third of the diameter, a in this case), $\Delta G(t)$ is inversely proportional to the square root of a and therefore the contribution of backlogged users becomes negligible when a is large. Thus, recalling the argument about the contribution of nodes n_i such that $t > d(n_0, n_1) + r_1 - r_0$ or $t < d(n_0, n_1) + r_1 - r_0 - 1$, for $a \to \infty$, we can state that $\lambda^{\rm end}(t)$ approaches a unit step multiplied by the rate G, that is:

$$\lambda^{\rm end}(t) \to \lambda^{\rm end}_\infty(t) = \begin{cases} G, & t \ge 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{for} \quad a \to \infty \ . \tag{10}$$

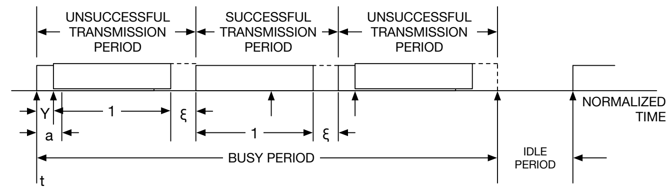

Figure 4. Sample case for 1-persistent CSMA: transmission periods, busy periods, and idle periods. (Adapted from [8].)

Based on these two considerations, we make the following two working assumptions:

- the rate $\lambda^{\rm st}(t)$ is constant during the packet transmission time, i.e., even though the spatial distribution is taken into account, it behaves as though nodes exhibit a common propagation delay equal to 1; along the lines of [8], this is equivalent to assuming that the vulnerable period (VP) has length 1 if $a \ge 1$;
- the expression for the average idle period time assumes the same form as in [8].

IV. Derivation of the proposed model

Based on the considerations developed in Section III-D, it becomes reasonable to derive an analytical model based on the one by Tobagi and Kleinrock. From now on we refer to the analysis in [8] with a few adjustments, and we assume $a \ge 1$, which is very often the case in underwater networks.

The throughput analysis makes use of renewal theory concepts. The throughput S can be expressed as a function of the offered traffic G as:

$$S(G) = GP_s(G) \ , \tag{11}$$

where P_s represents the success probability for a single packet transmission. As claimed in [8], a packet can be successful in two cases:

1) a users begins its Transmission Period (TP) when no other user is transmitting, and none of them begin their transmission during the VP of the transmission;
2) a packet arrives during another user's transmission and it is the only packet arriving in this period; moreover, its sender will be the only node beginning a transmission when the channel becomes idle, and no other transmission takes place during the VP of this new transmission. This is the situation depicted in Fig. 4 for the successful TP.

However, there is also another case where a packet transmission can be successful. If a packet in case 2) is not the only one, it might be successful because of the spatial distribution of the nodes: a packet by a node $n_0 = (r_0, \theta_0)$ could be successful if it begins its transmission at time t_0, when the channel is sensed idle, and all the other packets generated concurrently are received at the AP after $t_0 + r_0 + 1$, i.e., the transmissions by other nodes are received at the AP after the reception from n_0 has been completed. We approximate this probability as P_s. We will see that this approximation is quite accurate for a sufficiently large. Therefore, by defining $\bar{I}$, $\bar{B}$, $\bar{B}'$ as in [8] and letting $\bar{C} = \bar{I} + \bar{B}$ we have the following expression for P_s:

$$P_s(G) = \frac{\bar{I}}{\bar{C}} P_0 + \frac{\bar{B}'}{\bar{C}} \hat{q}_0 P_0 + \frac{\bar{B}'}{\bar{C}} (1 - \hat{q}_0) P_s(G) \ , \qquad (12)$$

where P_0 is the probability of having no arrivals during the VP and $\hat{q}_0$ is the probability of having no arrivals during the slot in which the packet is waiting for the channel to become idle. Note that, for simplicity, we have dropped the dependence of all terms on G. Eq. (12) yields

$$P_s = P_0 \frac{\bar{I} + \bar{B}'\hat{q}_0}{\bar{B} - \bar{B}'(1 - \hat{q}_0) + \bar{I}}. \qquad (13)$$

The length of the average idle period is found as

$$\bar{I} = \int_0^\infty \exp\left\{ - \int_0^t \lambda_\infty^{\text{end}}(u)\, \mathrm{d}u \right\} \mathrm{d}t = \frac{1}{G} \ , \qquad (14)$$

where we have considered $\lambda^{\text{end}}(t)$ in its asymptotic form $\lambda_\infty^{\text{end}}(t)$.

There remain to evaluate the other quantities in Eq. (11). Because $\lambda^{\text{st}}(t)$ is constantly equal to G in $t \in [0, 1]$, i.e., the VP has length 1, we have

$$P_0 = \exp\left\{ - \int_0^\infty \lambda_\infty^{\text{st}}(t) dt \right\} = e^{-G} \ . \qquad (15)$$

The distribution of the time of the last colliding transmission occurring during the VP is:

$$F_Y(t) = e^{-G(1-t)} \quad \text{for} \quad t \leq 1 \ , \qquad (16)$$

and

$$\bar{Y} = 1 - \frac{1}{G}\left(1 - e^{-G}\right) \ . \qquad (17)$$

The probability of having no arrivals during transmissions in a TP is

$$q_0 = e^{-2G}(1 + G) \ . \qquad (18)$$

The average duration of a TP is

$$\bar{T} = 1 + \bar{Y} + \bar{\xi} \ , \qquad (19)$$

where $\bar{\xi}$ is the average time that elapses at the AP from the end of the reception of the colliding transmission to the beginning of a new transmission due to a backlogged user. In the following, we assume $\bar{\xi}$ to be equal to the vulnerability period at the AP, i.e., $\bar{\xi} = 1$. As proven by the results in Section V, the impact of this approximation is negligible.

We can now derive the value of the busy period (BP) $\bar{B}$ (that counts the total duration of all TPs in a cycle), and the value of the BP $\bar{B}'$ (that does not include the intervals of duration ξ in each TP where the AP is not receiving. We have:

$$\bar{B} = \frac{2 + \bar{Y}}{q_0} \ , \qquad \bar{B}' = \frac{1 + \bar{Y}}{q_0} \ . \qquad (20)$$

Finally, $\hat{q}_0$ should be evaluated by considering the distribution function of $\hat{Z}$, that is the distribution function of the segment Z where the packet arrival occurred (its distribution is different than f_Z because of the residual time paradox and

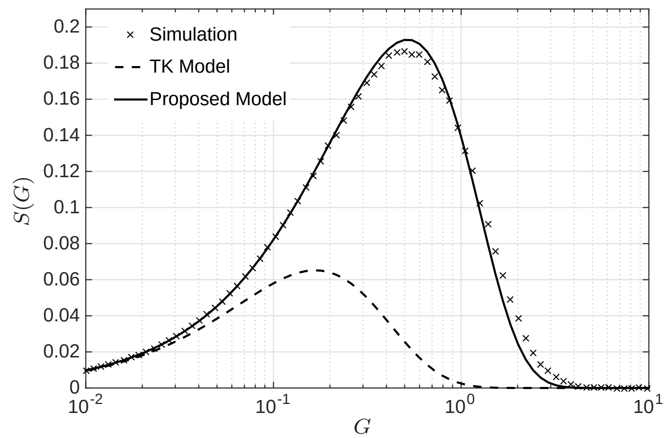

Figure 5. Comparison of simulation results of 1p-CSMA compared to the TK model and to our model for $a = 5$.

is given by the weighted distribution function of f_Z, that is $f_{\hat{Z}}(x) = x f_Z(x)/\bar{Z}$) [8]; this yields

$$\hat{q}_0 = \frac{e^{-2G}}{1 + \bar{Y}} \left[1 + \frac{3}{2} G \right] \ . \qquad (21)$$

Substituting (14), (15), (20) and (21) into (13), and the latter into (11), completes the derivation of the model.

V. Simulation results and model validation

We simulated the behavior of the system using OMNeT++ [10]. For the scenario presented in Section II, we set a number of hosts equal to $N = 100$. The main simulation parameters are a and the average interarrival time for each packet generated by a node, $\mu = \frac{1}{\lambda} = \frac{N}{G}$, where G is the aggregate packet generation rate. New arrivals are generated according to a Poisson process of rate λ. We have run simulations for different values of a: we noticed that the throughput rapidly converges to the same behavior for $a \geq 2$. This is due to the fact that the asymptotic behavior of the rates is achieved rapidly as a increases. The TK model is outperformed by the one introduced here even in the presence of the approximations in (9) and (10), and under our assumptions on $P_s(G)$ in (12) and on $\bar{\xi}$.

Fig. 5 shows $S(G)$ vs. G for 1p-CSMA as obtained by OMNeT++ simulations in the case $a = 5$, and compares it to the TK model and to our proposed model. We observe that the match between our model and the simulation is very good, whereas the TK model severely underestimated the maximum throughput (about $1/3$ of the value observed in the simulations) and also underestimates the value of G at which the maximum throughput is achieved.

Finally, Fig. 6 shows $S(G)$ vs. G for various values of a: for values of $a \geq 2$ the simulation curves become progressively harder to distinguish, and are very well approximated by the model. From this result, and from the fact that our model is independent of a, we infer that the influence of the propagation delay on $S(G)$ becomes progressively smaller as a increases. This can be explained by the fact that the process

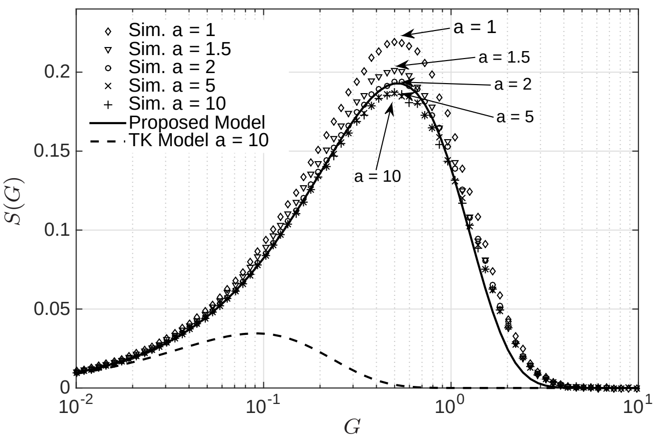

Figure 6. Comparison of simulation results of 1p-CSMA compared to the TK model and to our model.

of transmission successes and collisions at the access point achieves a stationary behavior as a increases, and the only factor that still counts is the normalized packet transmission time, which is identically 1.

VI. Conclusions

In this paper we have provided an approximate model for 1-persistent (1p) CSMA based on previous observations on non-persistent CSMA in [1]. We highlight that the 1p case is more complex due to the presence of backlogged users, that introduce memory and traffic correlation. However, we show that the evolution of the channel access rate over time has a regular behavior for $a \geq 2$, which makes it possible to approximate these rates as constant over time as in the model by Tobagi and Kleinrock (TK). We conclude that the assumptions of the TK model remain valid as an approximation, provided that the expression for the probability of success is modified to account for those transmission successes made possible by the large propagation delays, even in the presence of concurrent accesses.

This fundamental analysis is important for underwater communication networks, as in many cases CSMA has been part of the protocol stack in these scenarios. Our model has been compared against simulations for 1p-CSMA in several cases relevant to underwater networks, showing that it approximates simulations closely for a number of typical values of the propagation delay normalized to the packet transmission time.

Future work includes a better characterization of the time period between two subsequent transmission periods, and of the success probability in the presence of concurrent transmissions by backlogged users.

Acknowledgments

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