

Exact Resource Allocation for Weighted Proportional Fair Wireless Relay Networks

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Abstract—In this paper, we consider a relay-enabled wireless network and optimize the weighted proportional fair (wPF) bandwidth allocation by means of exact algorithms with linear complexity on the number of users and relays. Complex architectures typical of relay-enabled systems pose the need of developing efficient techniques that account for the intertwined nature of all the network agents, as resources must be split not only between users, but also between relays and relay-served users, altogether constrained by backhaul capacities and the traffic bottleneck present at the wired base stations. Here, traditional schemes for bandwidth allocation cannot be applied, as resources from one point of the network cannot be allocated regardless the allocation performed at other entities of the same network. Hence, we develop a compact weighted proportional fair resource management with very lightweight complexity, able to jointly allocate access and backhaul resources optimally in real time. We benchmark the results on network capacity and fairness with state-of-art proposals and show that the wPF exact algorithms proposed yield the best trade-off between capacity and fairness in relay networks.

I. INTRODUCTION

With the increasing densification of networks, the strategic use of wireless relays has emerged as a cost-effective solution to offload traffic, optimize network resource utilization, and enhance end-user connectivity quality. In present-day cellular infrastructures, a heterogeneous network of wired terrestrial base stations—primarily gNB s (Next Generation Node Bs), in the 3GPP 5G jargon—receives traffic from the core network and efficiently manages its distribution to end users. These gNB s not only provide service to users within their coverage area but can also establish high-capacity backhaul links with relays to redirect traffic and deliver significantly improved throughput to end users. To do so, specifications for 5G and beyond include, since ETSI/3GPP Release 17 [1], the possibility to use ‘sidelinks’ for device-to-device communications and for extending a gNB coverage through an intermediate node that reuses gNB frequency bands [2].

This study focuses on a heterogeneous relay-enabled network [3], consisting of fixed gNB s that deliver wireless service to both mobile users and intermediate relay nodes. Subsequently, these relay nodes deliver service to additional mobile users. Figure 1 provides an illustrative example of the reference scenario mentioned above.

At this stage, it is important to highlight that the complexity of relay architectures significantly complicates the analysis due to the intertwined interactions among all involved entities. First

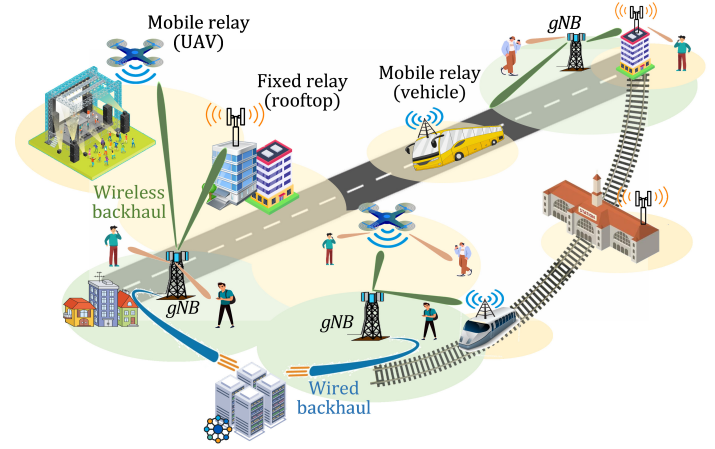


Figure 1: Reference scenario of a relay-enabled cellular network.

of all, it must be taken into account that wireless resource assignments must be performed rapidly to adapt to dynamic scenarios. Moreover, gNB resources must be allocated not only to directly served users but also shared with relays, thereby introducing interference since these relays may reuse wireless resources to serve their mobile users. In addition, gNB resource allocation is subject to backhaul capacity constraints, which must be carefully considered.

In this paper, we propose a mechanism for fair downlink rate allocation among mobile users. Given the high cost and limited availability of transmission resources, insufficient to serve all users at peak quality, a carefully designed resource allocation strategy is essential to maintain user satisfaction in the delivery of online services. In particular, we adopt a *weighted proportional fairness* (wPF) criterion [3], which seeks to allocate resources in a way that optimizes the overall system utility while ensuring an efficient yet equitable distribution of resources among users (i.e., any improvement for an individual user would entail a proportionally-weighted reduction for others, thus maintaining equilibrium across the network).

Although alternative fairness metrics exist, weighted proportional fairness provides a flexible balance between efficiency and fairness by allowing differentiated service levels across users. This is particularly relevant in practical systems where users or applications may have varying priorities or service agreements, and a strictly uniform allocation of weights (as, for instance, in max-min fairness) would be suboptimal or

economically inefficient. In such contexts, this becomes a suitable approach, as it enables differentiated resource allocation based on user priority or demand, while still guaranteeing that all users attain a minimum serviceable level, thereby aligning resource distribution with both efficiency and fairness objectives.

Our proposal is hence based on the rigorous analysis of the resource allocation problem in wireless relay networks, which allows us to formulate an exact solution for the problem. Most importantly, our solution is not only exact but also lightweight, as it scales linearly with the number of relays and customers. In contrast, legacy optimizers exhibit cubic complexity.

II. RELATED WORK

In recent years, there has been a growing body of research devoted to resource allocation in heterogeneous networks [3]. Here, we review relevant prior work, outline various research directions pursued in the literature, and highlight key aspects that distinguish our approach from existing studies.

The authors in [4] address a wireless downlink scenario enhanced by a single UAV, seeking to optimize the minimum average throughput experienced by users. In contrast to our work, they restrict their analysis to the single-relay case and do not consider the presence of multiple relays.

In [5], the authors investigate proportional and max-min fairness mechanisms in cognitive radio networks, where secondary users act as relays to guarantee acceptable user rates. However, their analysis requires solving nonconvex optimization problems, which limits the ability to obtain optimal solutions efficiently. In contrast, our approach delivers exact solutions with linear-time complexity. In [6], the authors consider a scenario that resembles ours. Nevertheless, their analysis relies on restrictive assumptions concerning resource allocation and neglects both inter-cell interference and the interference between gNB s and relays. Under these simplifications, they propose a suboptimal heuristic and demonstrate its potential to improve fairness. In [7], the authors study satellite-terrestrial relay networks, with the aim of maximizing user rates under fairness constraints for spectrum allocation and user association. However, their approach relies on heuristic methods, which remain inherently suboptimal. In [8], the authors focus on relay selection in dense heterogeneous networks to achieve load balance fairness, although their approach is primarily tailored to device-to-device communication scenarios. In contrast to both [7] and [8], our work explicitly guarantees a minimum service level for users, which is not ensured in those previous approaches.

In [9], the authors consider a scenario that closely resembles ours. However, unlike their work, we adopt a weighted proportional fairness criterion rather than max-min fairness, which entails fundamentally different analytical and algorithmic approaches. Indeed, while the max-min fairness problem is linear, the wPF formulation is inherently nonlinear. As a result, the solution techniques used in the max-min case are not applicable or adaptable to our setting.

Our work: In this paper, we propose a resource allocation framework based on weighted proportional fairness for wireless relay networks, which enables the joint distribution of resources among mobile users and multiple relays. The design explicitly incorporates wired and wireless backhaul constraints, which pose coupling limitations that render conventional scheduling strategies inapplicable. Our algorithm computes the exact solution to the underlying optimization problem, enabling efficient computation. Given that the proportional fairness formulation is inherently non-linear, this solution is derived by exploiting the convex structure of the problem. Indeed, we leverage the analytic structure of the problem to formulate algorithms that reach the optimum solution with linear complexity, which is much lighter than what doable with legacy optimizers for convex functions, which run in cubic time.

While prior work has addressed fairness and resource allocation in cellular networks, most existing methods either rely on computationally intensive solvers or lack end-to-end modeling of relay-assisted architectures with constrained backhaul. Moreover, few proposals offer practical mechanisms to differentiate users by service class or application demands. In this work, we fill this gap by proposing an exact, lightweight algorithm for weighted proportional fairness that supports user prioritization and is applicable to realistic wireless relay networks, e.g., 5G and beyond 5G networks using sidelinks with multiple relays. Our approach achieves the same optimal allocation as solver-based methods, but with linear complexity, and significantly outperforms fairness-oriented heuristics and simplified approximations in realistic urban deployments, as demonstrated in our experimental evaluation.

III. SYSTEM MODEL

We assume a relay-enabled cellular network composed by a set of gNB s $\mathcal{G}$ and a set of relays $\mathcal{R}$ that provide access service to a set of mobile users $\mathcal{U}$. While gNB s are wired to the backhaul network, relays use a wireless backhaul link to offload traffic from the gNB they are attached to.

The network of relays is hybrid, meaning that relays may be of different types and functionalities, e.g., fixed relays at rooftops to extend coverage, or mobile relays in vehicles (buses, trains) to provide connectivity to passengers, or extremely-mobile relays like UAVs to adapt and optimize the network on demand, etc. Finding the optimal attachment policy between gNB s, relays and users has already been solved in previous works [10], depending on the scenario, and is out of the scope of this paper. What matters here is that, at the time of bandwidth allocation in the network, each relay is attached to a corresponding gNB with a backhaul link. Also users are attached to some serving base station (either a gNB or a relay). Hence, the Channel State Information (CSI) of each access and backhaul link is known by the orchestrators (in this case, the gNB s) and optimal bandwidth allocation is performed based on such CSI.

Channels. For backhaul connections, we assume that gNB s perform 3D-beamforming over wireless links, which allow for

high-capacity links and simplified interference management, thanks to directionality. Hence, gNB s provide backhaul service to the relays while both gNB s and the relays provide access network service to the users. Specifically, each access cell (from gNB s or from relays) performs an OFDMA service, so that users inside the same cell do not suffer intra-cell interference.

Interference. However, users from different gNB - or relay-served cells might suffer inter-cell interference. In addition, backhaul links might interfere among themselves if their antennas point to different sources of backhaul beams, and even also served users might interfere with such backhaul connections, if they reuse the same band.

Hence, depending on the nature of each link (access or backhaul), the SINR experienced must be considered. We denote by $\gamma_{s,x}$ the SINR experienced between a BS s (either a gNB or a relay r) and a node x (either a relay r or a user u), which captures the signal strength of wireless backhaul links as well as user access links. Such SINR must be properly computed according to the CSI from the considered application scenario, matching the particular relay network features and requirements.

Bandwidth management. The available channel bandwidth of the gNB access channels, relay access channels and backhaul channels are B_g , B_R and B_B , respectively. Moreover, in order to guarantee the cellular service, each cell user requesting service and each backhaul relay must be allocated at least a portion of the available resources. Concretely, gNB -served users get a minimum of $B_g^{\min}$ bandwidth; relay-served users get a minimum of $B_R^{\min}$ bandwidth; and backhaul relays get a minimum of $B_B^{\min}$ resources. Additionally, each gNB $g \in \mathcal{G}$ wired to the core network receives traffic from the Internet at a maximum capacity rate of Θ_g , which acts as the bottleneck of the service passing through that gNB .

IV. THE RESOURCE ALLOCATION PROBLEM

We aim to maximize the global network performance in terms of fairly allocated throughput. Here, all the involved agents (gNB s, relays and users) are intertwined, as the interference management involves different types of wireless channels colliding. Hence, at the moment of the bandwidth allocation, the network disposes of the CSI necessary to know the quality of the channels, depending on the interference observed. As a result, each gNB solves the optimal bandwidth allocation in parallel, with a convex program that we formulate next.

Bandwidth allocation in relay systems. In particular, each gNB $g \in \mathcal{G}$ must perform bandwidth allocation to split backhaul resources among the served relays, split gNB -UE access resources among the gNB -attached users, $\mathcal{U}_g$, and the attached relays must split relay-UE access resources among the users attached to each relay $r \in \mathcal{R}$, $\mathcal{U}_r$. In relay systems as the one we address in this paper, all these bandwidth allocations are intertwined (relay-UE allocation depends on the wireless backhaul bottleneck; gNB -relay allocation depends on the set of attached relays to the gNB , $\mathcal{R}_g$ and the number of final users of each relay; and gNB -UE access resources depend

also on the backbone service that the gNB obtains from the core network). Hence, the bandwidth allocation problem needs a new approach that cannot be addressed with traditional schedules. As a result, we formulate the bandwidth allocation optimization problem for relay networks as a convex program in Eq. (1).

Network utility. The goal of this work is to optimize network performance by optimizing a weighted proportional fair allocation of bandwidth resources (referred as the **Exact wPF** scheme). User weights are denoted as w_u .

In Eq. (1), the utility function U_g of each gNB g is the weighted proportional fairness metric.

$$\begin{cases} \max U_g = \sum_{u \in \mathcal{U}_g \cup \bigcup_{r \in \mathcal{R}_g} \mathcal{U}_r} w_u \log(\theta_u); \\ \text{s.t.:} \\ b_r \geq B_B^{\min}, & \forall r \in \mathcal{R}_g; \\ \sum_{r \in \mathcal{R}_g} b_r = B_B; \\ \theta_r \leq b_r \log_2(1 + \gamma_{g,r}), & \forall r \in \mathcal{R}_g; \\ b_u \geq B_g^{\min}, & \forall u \in \mathcal{U}_g; \\ \sum_{u \in \mathcal{U}_g} b_u = B_g; \\ b_u \geq B_R^{\min}, & \forall u \in \bigcup_{r \in \mathcal{R}_g} \mathcal{U}_r; \\ \sum_{u \in \mathcal{U}_r} b_u = B_R, & \forall r \in \mathcal{R}_g; \\ \theta_u \leq b_u \log_2(1 + \gamma_{s,u}), & \forall s \in \{g\} \cup \mathcal{R}_g, \forall u \in \mathcal{U}_s; \\ \sum_{u \in \mathcal{U}_r} \theta_u \leq \theta_r, & \forall r \in \mathcal{R}_g; \\ \sum_{u \in \mathcal{U}_g} \theta_u + \sum_{r \in \mathcal{R}_g} \theta_r \leq \Theta_g. \end{cases} \quad (1)$$

Decision variables. In Eq. (1), b_r is the share of total bandwidth that the gNB g allocates to the relay r , θ_r is the backhaul throughput for the relay r , b_u is the share bandwidth that a BS s (either the gNB or a relay) allocates to a user u , and θ_u is the access service throughput for user u .

Constraints. For backhaul relay allocations, the first constraint guarantees that each relay obtains a minimum of $B_B^{\min}$ bandwidth, the second constraint states that the aggregated share bandwidth of relays must adjust to the whole channel capacity, and the third constraint restricts the throughput allocated to relays to be at most equal to the Shannon capacity. For gNB -served users, the forth constraint guarantees a minimum $B_g^{\min}$ bandwidth for each user and the fifth constraint states that the aggregate share bandwidth of these users must adjust to the whole channel capacity, while the sixth and seventh constraints state the analogous conditions for relay-served users. The eighth constraint restricts the throughput allocated to users (either served by the gNB or by some relay) to not exceed the Shannon capacity, as well as the ninth constraint, which restricts the throughput allocated to each relay to not exceed the share of the backhaul capacity received. Finally, the last constraint ensures that the aggregated throughput serviced by the gNB must not exceed backbone capacity, acting as network bottleneck.

Complexity. The resource allocation problem under consideration is convex-although non-linear due to the nature of the wPF objective, and is therefore solvable in polynomial time using standard convex optimization tools such as interior-point methods [11]. However, these methods exhibit cubic

time complexity with respect to the number of users, i.e., $\mathcal{O}(|\mathcal{U}|^3)$ [12], which can become computationally infeasible for real-time applications. This is particularly problematic in scenarios involving high user density or dynamic relay-based infrastructures, where on-the-fly scheduling is required.

In contrast, this paper introduces an exact algorithmic solution based on the Karush–Kuhn–Tucker (KKT) conditions [13], which enables us to bypass heavy numerical solvers and compute the optimal solution analytically with significantly lower complexity. With that exact algorithm, the complexity of computing the optimal bandwidth allocation for a given gNB g is linear in the number of associated users and relays, i.e., $\mathcal{O}(|\mathcal{U}_g| \cdot |\mathcal{R}_g|)$ in the worst-case scenario. This result is a key contribution of our work, as it allows for real-time optimal resource allocation in dense and dynamic wireless relay networks, where computational efficiency is critical.

V. THE EXACT RESOURCE ALLOCATION FOR WEIGHTED PROPORTIONAL FAIRNESS

We present an exact, low-complexity solution to the wPF allocation problem introduced in Eq. (1). While the problem is convex, its non-linear nature makes conventional solvers inefficient for real-time applications. By deriving the KKT conditions [13], we obtain a closed-form algorithm that avoids iterative optimization and delivers the exact optimal solution with significantly lower complexity. Our method addresses two key scenarios:

- The *settling* case, where a gNB serves users without relays.
- The *plenary* case, which involves multiple relays and coordinated access/backhaul resource sharing.

The solution to the *settling* case serves as a building block for solving the more general *plenary* case efficiently. This modular approach seeks both scalability and precision, in order to enable exact and real-time allocation even in dense or dynamic relay-enabled networks.

A. The settling scenario: One base station without relays

In this scenario, there are no relays. Then, gNB g only needs to manage resources to be split among gNB -served users, i.e., the set $\mathcal{U}_g$. Hence, we solve the following CP:

$$\begin{cases} \max & U_g = \sum_{u \in \mathcal{U}_g} w_u \log(\theta_u); \\ & b_u \geq B_g^{\min}, \quad \forall u \in \mathcal{U}_g; \\ & \sum_{u \in \mathcal{U}_g} b_u = B_g; \\ & \theta_u \leq b_u \log_2(1 + \gamma_{g,u}), \quad \forall u \in \mathcal{U}_g; \\ & \sum_{u \in \mathcal{U}_g} \theta_u \leq \Theta_g. \end{cases} \quad (2)$$

Denote as $x = [\{b_u\}_{u \in \mathcal{U}_g}, \{\theta_u\}_{u \in \mathcal{U}_g}] \in \mathbb{R}^{2|\mathcal{U}_g|}$ the vector of decision variables. Then, the KKT functions are:

$$f(x) = - \sum_{u \in \mathcal{U}_g} w_u \log \theta_u; \quad (3)$$

$$g_u(x) = B_g^{\min} - b_u, \quad \forall u \in \mathcal{U}_g; \quad (4)$$

$$g_{|\mathcal{U}_g|+u}(x) = \theta_u - b_u \log_2(1 + \gamma_{g,u}), \quad \forall u \in \mathcal{U}_g; \quad (5)$$

$$g_{2|\mathcal{U}_g|+1}(x) = \sum_{u \in \mathcal{U}_g} \theta_u - \Theta_g; \quad (6)$$

$$h(x) = \sum_{u \in \mathcal{U}_g} b_u - B_g. \quad (7)$$

The KKT conditions state that if we find a vector $x^* \in \mathbb{R}^{2|\mathcal{U}_g|}$ that is feasible (i.e., it satisfies the constraints of the problem), and multipliers ν and $\mu_i \geq 0, \forall 1 \leq i \leq 2|\mathcal{U}_g| + 1$, not all of them null, so that the following equations hold, then x^* is the optimal solution:

$$\mathbf{0} = \nabla f(x^*) + \sum_{i=1}^{2|\mathcal{U}_g|+1} \mu_i \nabla g_i(x^*) + \nu \nabla h(x^*); \quad (8)$$

$$h(x^*) = 0 \text{ and } \mu_i g_i(x^*) = 0, \quad \forall 1 \leq i \leq 2|\mathcal{U}_g| + 1. \quad (9)$$

Hence, in what follows we describe how to find the exact solution.

Recall that we aim to maximize the network weighted proportional fairness metric, $f(x) = - \sum_{u \in \mathcal{U}_g} w_u \log(\theta_u)$. Then, the KKT conditions for this case are:

$$-\mu_u - \log_2(1 + \gamma_{g,u}) \cdot \mu_{|\mathcal{U}_g|+u} + \nu = 0, \quad \forall u \in \mathcal{U}_g; \quad (10)$$

$$\mu_{|\mathcal{U}_g|+u} + \mu_{2|\mathcal{U}_g|+1} - w_u/\theta_u = 0, \quad \forall u \in \mathcal{U}_g; \quad (11)$$

$$\mu_u \cdot (B_g^{\min} - b_u) = 0, \quad \forall u \in \mathcal{U}_g; \quad (12)$$

$$\mu_{|\mathcal{U}_g|+u} \cdot (\theta_u - b_u \cdot \log_2(1 + \gamma_{g,u})) = 0, \quad \forall u \in \mathcal{U}_g; \quad (13)$$

$$\mu_{2|\mathcal{U}_g|+1} \cdot \left(\sum_{u \in \mathcal{U}_g} \theta_u - \Theta_g \right) = 0; \quad (14)$$

$$\sum_{u \in \mathcal{U}_g} b_u - B_g = 0. \quad (15)$$

With unbounded capacity Θ_g . If Θ_g is unbounded, Eq. (14) requires that $\mu_{2|\mathcal{U}_g|+1} = 0$. Hence, according to Eq. (11), $\mu_{|\mathcal{U}_g|+u} = \frac{w_u}{\theta_u}, \forall u \in \mathcal{U}_g$.

Now, $\forall u \in \mathcal{U}_g$, let $\mu_u = 0$, $b_u = w_u B_g / \sum_{u \in \mathcal{U}_g} w_u$ and $\theta_u = w_u B_g / \sum_{u \in \mathcal{U}_g} w_u \log_2(1 + \gamma_{g,u})$. Hence, from Eq. (10), $\nu = \sum_{u \in \mathcal{U}_g} w_u / B_g$.

Since the found a feasible solution x that accomplishes all KKT conditions, this is the optimal solution. Note that, without the wired backhaul constraint, all users would obtain an amount of resources proportional to their weight.

With limited capacity Θ_g . In the case where the backhaul capacity Θ_g is limited, the resource allocation obtained under the assumption of unbounded capacity may exceed this constraint—i.e., the total allocated user throughput $\sum_{u \in \mathcal{U}_g} \theta_u$ may exceed Θ_g . To address this, we need to adjust the individual user throughputs downward, in a way that minimally degrades the global utility, optimally. Since the utility function is defined as a weighted logarithmic sum, which is a strictly concave and differentiable function, its derivative with respect to each user throughput is w_u/θ_u . This derivative indicates the marginal utility gain of allocating an additional unit of rate to user u . To minimize the loss in total utility when reducing rates, we need to decrease the throughput of those users whose marginal contribution is lowest, i.e., those with smallest w_u/θ_u . Now, we define the rate-to-weight ratio as θ_u/w_u and, thus, users with the highest rate-to-weight ratios are the ones contributing the least per unit of throughput to the total utility and must be prioritized for reduction.

By means of Algorithm 1, we iteratively reduce the throughput of users with maximum rate-to-weight ratio, bringing them down toward the next lower rate-to-weight ratio in the system. This approach ensures that all reductions are done

Algorithm 1 Fair rate adjustment based on rate-to-weight ratio

Require: $n, \{x_i\}_{i=1}^n, \{\omega_i\}_{i=1}^n, X$.

- 1: **while** $\sum_{i=1}^n x_i > X$ **do**
- 2: $\rho_i \leftarrow x_i/\omega_i, \forall 1 \leq i \leq n$.
- 3: $\rho_{max} \leftarrow \max_{1 \leq i \leq n} \{\rho_i\}$.
- 4: $\mathcal{I} \leftarrow \{i \mid \rho_i = \rho_{max}\}$.
- 5: $\rho_{next} \leftarrow \max \{\rho_i \mid i \notin \mathcal{I}\}$.
- 6: $\Delta \leftarrow \sum_{i \in \mathcal{I}} (x_i - \omega_i \cdot \rho_{next})$.
- 7: **if** $\sum_{i=1}^n x_i - \Delta \leq X$ **then**
- 8: $E \leftarrow \sum_{i=1}^n x_i - X$.
- 9: $x_i \leftarrow x_i - E \cdot (\omega_i / \sum_{j \in \mathcal{I}} \omega_j), \forall i \in \mathcal{I}$.
- 10: **else**
- 11: $x_i \leftarrow \omega_i \cdot \rho_{next}, \forall i \in \mathcal{I}$.
- 12: **end if**
- 13: **end while**

equitably and optimally, avoiding unnecessary cuts in high-utility users. The process is as follows: Firstly, identify the group of users with the current highest rate-to-weight ratio $\rho_{max} = \max_i \{\theta_i/\omega_i\}$ (steps 2-3). Secondly, identify the next lower rate-to-weight ratio (i.e., the second-highest in the system), to which the current group could be reduced (steps 4-5). Then, compute whether reducing all current high users rate-to-weight ratio down to that next level is enough to meet the capacity constraint Θ_g and, if so, calculate the exact target rate to reduce them just enough to meet Θ_g (steps 6-11). Finally, apply the reduction, and repeat if necessary. This is implemented in Algorithm 1, where n is the number of users, $\{x_i\}_{i=1}^n = \{\theta_u\}_{u \in \mathcal{U}_g}$ are the current user throughputs, $\{\omega_i\}_{i=1}^n = \{\omega_u\}_{u \in \mathcal{U}_g}$ are the user weights and $X = \Theta_g$ is the total backhaul capacity constraint. The algorithm continues until $\sum_{u \in \mathcal{U}_g} \theta_u = \Theta_g$, ensuring that the minimal possible utility loss is incurred.

It can be noted that the key of this method lies in exploiting the structure of the logarithmic utility: since users with higher throughput per weight are less sensitive to reductions, the algorithm strategically balances rate cuts to maximize fairness-adjusted efficiency under a hard backhaul limit.

B. The plenary scenario: General case with relays

We now address the general case described in Eq. (1), where a gNB allocates resources not only to directly attached users but also to multiple relays, each of which serves its own set of users. As before, we distinguish between scenarios where the backhaul capacity Θ_g is either unbounded or limited.

With unbounded capacity Θ_g . When Θ_g is unbounded, the resource allocation to users directly served by the gNB is decoupled from the allocation to relay nodes. As such, we can independently apply the *settling*-case solution from Section V-A to determine the optimal bandwidth allocation for gNB -attached users.

Then, for each relay $r \in \mathcal{R}_g$, we similarly compute the optimal resource allocation under the assumption of unbounded wireless backhaul throughput θ_r . That is, relay-served user

rates, now denoted as $\theta_{r,u}$ for convenience, are limited solely by the relay r 's allocated bandwidth and the SINR of the access links (r, u) , following the same method used for gNB users. The total backhaul throughput required by relay r is then $\theta_r = \sum_{u \in \mathcal{U}_r} \theta_{r,u}$.

To support this throughput, each relay r requires a share of gNB -to-relay bandwidth b_r of $b_r = \max(B_B^{\min}, \sum_{u \in \mathcal{U}_r} \theta_{r,u} / \log_2(1 + \gamma_{g,r}))$.

If the total backhaul bandwidth required by all relays fits within the gNB 's available backhaul capacity B_B , i.e., $\sum_{r \in \mathcal{R}_g} b_r \leq B_B$, then the remaining bandwidth can be assigned arbitrarily (e.g., added to one relay), and the solution is complete.

However, if $\sum_{r \in \mathcal{R}_g} b_r > B_B$, then we exceed the available resources and must proceed with a more careful redistribution, following Algorithm 2, as we describe next.

In this case, relays r are initially assigned the minimum bandwidth, i.e., $b_r = B_B^{\min}$, and their backhaul throughputs are computed accordingly: $\theta_r = b_r \log_2(1 + \gamma_{g,r})$ (step 1). Next, within Algorithm 2, we recursively reallocate backhaul bandwidth among relays in a utility-optimal fashion. This procedure is hierarchical:

- Each relay r applies the *settling*-case solution (from Section V-A) locally to optimize its users' access rates under the current backhaul rate limit θ_r (step 2).
- Then, the gNB globally adjusts the rate limit values θ_r by incrementally increasing the rate of users with the highest marginal utility across the whole relay network, i.e., those with the lowest rate-to-weight ratio θ_u/ω_u , and only until their marginal utility does not equals the marginal utility of other users or the Shannon limit is met. If any of those circumstances occurs, and further unassigned backhaul rate is available, the process continues with a new group of users experiencing the highest marginal utility and not being already at their Shannon limit. In the process of incrementing rate limits, the allocation to user u has to be done proportionally to its weight w_u , so that the marginal utilities of all the user in the group remain equal while increasing the rate limit.

This recursive trimming mechanism maximizes the total network utility while respecting bandwidth constraints. Importantly, because the utility function is a sum of weighted logarithms (concave and differentiable), it ensures that throughput increases benefit the users who contribute most to global utility per additional unit of rate.

To do so, Algorithm 2 follows these steps:

- 1) Identify the minimum rate-to-weight ratio across all users in relay-served links who have not yet reached their Shannon capacity (step 5).
- 2) Define the set of relay access links with this minimum ratio, denoted $\mathcal{L}_m$ (step 6), and attempt to increase their rates up to the next higher rate-to-weight level in the system (step 7).
- 3) Compute the actual maximum level reachable rate-to-weight ratio (ρ_{targ}) without exceeding the physical capacity constraints, i.e., the Shannon limit (step 8).

Algorithm 2 Exact wPF utility-aware allocation with relays

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1:  $b_r \leftarrow B_g^{\min}$  and  $\theta_r \leftarrow b_r \log_2(1 + \gamma_{g,r})$ ,  $\forall r \in \mathcal{R}_g$ .
2: Derive  $\{\theta_u\}_{u \in \mathcal{U}_g}$  and  $\{\theta_{r,u}\}_{u \in \mathcal{U}_r}$ ,  $\forall r \in \mathcal{R}_g$  by solving optimal resource allocation for  $gNB$   $g$  assuming unbounded  $\Theta_g$  and assuming also  $\forall r \in \mathcal{R}_g$  a wireless backhaul limitation of  $\theta_r$ , as detailed in Subsection V-A.
3:  $\delta \leftarrow 1$ .
4: while  $\delta = 1$  do
5:    $\rho_{min} \leftarrow \min \left\{ \frac{\theta_{r,u}}{w_u} \mid \theta_{r,u} < b_u \log_2(1 + \gamma_{r,u}), r \in \mathcal{R}_g, u \in \mathcal{U}_r \right\}$ .
6:    $\mathcal{L}_m \leftarrow \left\{ (r, u) \in \mathcal{R}_g \times \bigcup_{r \in \mathcal{R}_g} \mathcal{U}_r \mid \frac{\theta_{r,u}}{w_u} = \rho_{min} \right\}$ .
7:    $\rho_{next} \leftarrow \min \left\{ \frac{\theta_{r,u}}{w_u} \mid \frac{\theta_{r,u}}{w_u} > \rho_{min}, r \in \mathcal{R}_g, u \in \mathcal{U}_r \right\}$ .
8:    $\rho_{targ} \leftarrow \min(\rho_{next}, \min_{(r,u) \in \mathcal{L}_m} b_u \log_2(1 + \gamma_{r,u}) / w_u)$ .
9:    $\bar{\mathcal{U}}_r \leftarrow \{u \in \mathcal{U}_r \mid (r, u) \in \mathcal{L}_m\}$ ,  $\forall r \in \mathcal{R}_g$ .
10:   $\delta \leftarrow \min \left( 1, \frac{B_g - \sum_{r \in \mathcal{R}_g} b_r}{(\rho_{targ} - \rho_{min}) \cdot \sum_{r \in \mathcal{R}_g} \frac{\sum_{u \in \bar{\mathcal{U}}_r} w_u}{\log_2(1 + \gamma_{g,r})}} \right)$ .
11:   $b_r \leftarrow b_r + \delta (\rho_{targ} - \rho_{min}) \frac{\sum_{u \in \bar{\mathcal{U}}_r} w_u}{\log_2(1 + \gamma_{g,r})}$ ,  $\forall r \in \mathcal{R}_g$ .
12:   $\theta_r \leftarrow b_r \log_2(1 + \gamma_{g,r})$ ,  $\forall r \in \mathcal{R}_g$ .
13:   $\theta_{r,u} \leftarrow \theta_{r,u} + \delta (\rho_{targ} - \rho_{min}) w_u$ ,  $\forall (r, u) \in \mathcal{L}_m$ .
14: end while
15: if  $\sum_{u \in \mathcal{U}_g} \theta_u + \sum_{r \in \mathcal{R}_g} \theta_r > \Theta_g$  then
16:   Apply Algorithm 1 to  $n = |\mathcal{U}_g| + \sum_{r \in \mathcal{R}_g} |\mathcal{U}_r|$ ,
    $\{x_i\}_{i=1}^n = \{\theta_u\}_{u \in \mathcal{U}_g} \cup \bigcup_{r \in \mathcal{R}_g} \{\theta_{r,u}\}_{u \in \mathcal{U}_r}$ ,  $\{\omega_i\}_{i=1}^n = \{p_u\}_{u \in \mathcal{U}_g} \cup \bigcup_{r \in \mathcal{R}_g} \mathcal{U}_r$ ,  $X = \Theta_g$ .
17: end if

```

- 4) Determine how much to increase the current users' rates, gathered in $\bar{\mathcal{U}}_r$ (step 9), using a parameter $\delta \in [0, 1]$, which represents the largest feasible increase proportion (step 10).
- 5) Update all involved user throughputs and recompute required relay backhaul bandwidths (steps 11–13).
- 6) Repeat the process until all feasible increases have been exhausted.

This algorithm effectively equalizes marginal utility across users, progressively assigning resources where they produce the highest utility gain, and adapting dynamically to capacity limitations.

With limited capacity Θ_g . Now, if the total user throughput, counting both gNB -attached and relay-attached users, still exceeds the gNB 's backhaul capacity Θ_g , a final adjustment step is needed. To that end, we apply Algorithm 1 globally to all user throughputs (both θ_u from gNB and $\theta_{r,u}$ from relays), using their respective weights, and reduce them until the total becomes Θ_g (step 16). This final step guarantees that the solution respects the hard end-to-end constraint while retaining optimality in the sense of weighted proportional fairness.

VI. EXPERIMENTAL EVALUATION

We now present the experimental evaluation of our weighted Proportional Fairness (wPF) resource allocation scheme. We benchmark our proposed algorithm—referred to as **Exact**

TABLE I: EVALUATION PARAMETERS

Parameter	Value/Description
Carrier frequency Bands, f_g, f_R	n3: 1.8 GHz, n7: 2.6 GHz
Tx power, P_{Tx}^g, P_{Tx}^r	40 dBm, 23 dBm
Thermal Noise Power	-174 dBm/Hz
Ground path loss exponent	3
Aerial relays channel model	LoS/NLoS dense networks [14]
LoS/NLoS excess path-loss (aerial)	1.6 dB, 23 dB [14]
Environment (aerial relay), a, b	12.08, 0.11 [14]
Channel bandwidths, B_g, B_g, B_R	50 MHz, 50 MHz, 50 MHz
Min. bandwidths, $B_g^{\min}, B_g^{\min}, B_R^{\min}$	360 kHz, 360 kHz, 360 kHz
gNB backhaul rate, Θ_g	1.5 Gbps
Cell selection (relays and users)	Best SINR [15]
Urban area size	20 km ²
User and relay placement	Uniform
Aerial relays height range	[40, 300] m
Instances of simulations	1000
Simulation tool	MATLAB R2025a

wPF—against three comparison baselines: Solver wPF, MaxMin, and Light wPF.

- Solver wPF employs a standard convex solver to obtain the wPF solution, with a computational complexity that grows cubically with the number of users.
- MaxMin implements the fairness-oriented approach from [9], aiming to maximize the minimum user rate at the expense of total network throughput.
- Light wPF is a wPF-inspired lightweight heuristic, designed to further reduce complexity compared to Exact wPF. Indeed, Light wPF needs a number of operations that grows linearly just with the number of users, in contrast to the linear dependence of Exact wPF on the number of users *and* relays. It follows a simplified allocation strategy: (i) the gNB distributes its backhaul resources among served relays in proportion to the amount of end-users attached to each relay; (ii) both the gNB and the relays allocate their available bandwidth among their served users in proportion to their user weights, following the wPF principle; (iii) if the aggregate throughput of users served by a relay exceeds its allocated backhaul capacity, their rates are scaled down proportionally to their initially assigned values to meet the constraint; (iv) if the total aggregate rate across all users (gNB - and relay-served) exceeds the gNB 's wired backhaul limit, all user rates are again scaled down proportionally. Note that when there are none or one relays, and backhaul constraints are relaxed, Light wPF and Exact wPF match.

Our simulation setup considers an urban deployment covering 20 km². Relays are modeled with aerial positioning, including both line-of-sight (LoS) and non-line-of-sight (NLoS) conditions typical of dense cellular environments, following proven path-loss models from related studies such as [10], [14], based on ITU recommendations [16]. Such models take into account environmental parameters such as the number of buildings, a large number of obstructions, and buildings' height distribution, as well as a stochastic model for LoS or NLoS channel fading and excess path-loss. Wireless backhaul

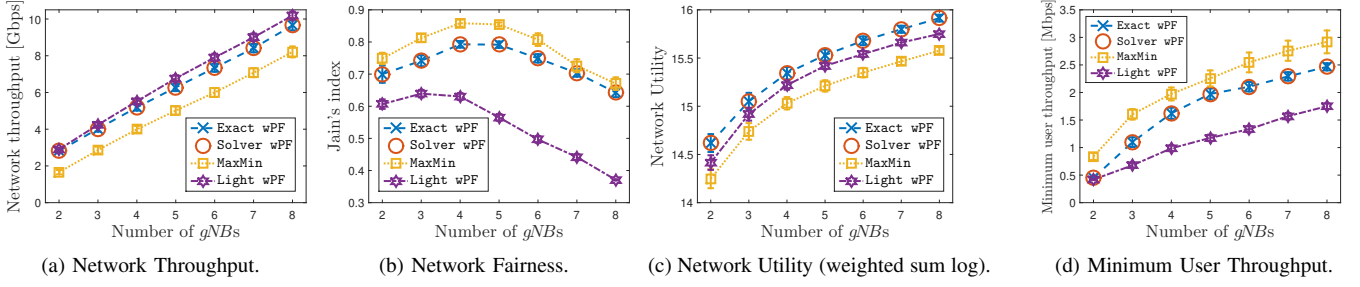


Figure 2: Wireless relay network with 5 relays per gNB and $U = 1000$ users. Scenario: VALIDATION.

links are established via 3D-beamforming between gNB s and relays with a half power beam-width of 65 degrees. Relays get attached to that gNB with better Signal-to-Noise Ratio (SINR). Also, users get access service from the gNB or relay providing best SINR, as in standard 3GPP networks cell selection protocols [15]. Table I summarizes the parameters used, consistent with dense 5G network deployments. Transmission power is set to 40 dBm for gNB s and 23 dBm for relays, using standardized 5G bands (n3 at 1.8 GHz for gNB access and backhaul; n7 at 2.6 GHz for relay access), each with a bandwidth of 50 MHz. Each gNB has a wired backhaul bottleneck capped at 1.5 Gbps. All simulations are run over 1000 independent network instances using MATLAB R2025a. In the figures, errorbars represent 95% confidence intervals.

To better illustrate the robustness and applicability of our scheme, we test it under two distinct scenarios: VALIDATION and REALISTIC.

The VALIDATION scenario. In this scenario, we validate the optimality of our Exact wPF scheme by comparing it to the Solver wPF implementation. The goal is to demonstrate that our lightweight algorithm achieves the same performance as the convex solver, but with significantly lower complexity—linear in the number of users and relays per gNB , compared to the cubic time required by standard solvers. We run experiments on uniformly-random generated network topologies with variable numbers of relays, using 1000 users and 5 relays per gNB on average. User weights are drawn from a Gaussian distribution with 20% variability (σ/μ) and normalized. Hence, this scenario is fully randomized, to capture the impact of any kind of topology, deployment, and user type distribution.

Figure 2 presents the results for several key performance indicators: network throughput, Jain’s fairness index, network utility, and minimum user throughput. As shown in Figure 2a, the Exact wPF scheme achieves the same throughput performance as Solver wPF, confirming the correctness of our solution. Notably, both wPF schemes significantly outperform the MaxMin scheduler in terms of capacity, validating the benefits of proportional fairness in balancing efficiency and fairness.

Although MaxMin slightly improves fairness (Figure 2b), the gain is marginal—less than 5 points in Jain’s index—compared to the notable throughput advantage achieved by Exact wPF. Figure 2c shows that Exact wPF reaches the same utility values as Solver wPF, which is the primary op-

timization objective of the algorithm. Meanwhile, the MaxMin scheme performs worse in utility since it prioritizes the most disadvantaged users at the expense of overall performance.

Eventually, Figure 2d highlights that although MaxMin achieves the highest minimum user throughput, the Exact wPF solution maintains a very competitive lower-bound performance, ensuring that no users experience outage while significantly improving total throughput.

Figures 2a to 2d also show results for Light wPF. Although such a simplified scheme is able to achieve the highest throughput, slightly higher than with Exact wPF (cf. Figure 2a), it clearly incurs a significant fairness degradation (cf. Figure 2b). This leads to both lower network utility – hence it is sub-optimal – although still better than with MaxMin (cf. Figure 2c) and, most importantly, it provides users with much lower rate guarantees (cf. Figure 2d).

In summary, this validation confirms that: (i) Exact wPF delivers optimal performance, matching Solver wPF across all metrics, while Light wPF is far from achieving optimal results; (ii) Exact wPF achieves the optimal with drastically reduced complexity, with respect to a legacy convex solver (Solver wPF); and (iii) it maintains a highly favorable trade-off between throughput and fairness, increasing network capacity by several Gbps while preserving acceptable fairness and protecting low-rate users from service degradation.

The REALISTIC scenario. For the Realistic scenario, we use the cellular layout of Leganés, a city near Madrid, Spain, with an area similar to the one of the validation scenario described before ($\sim 20 \text{ km}^2$), and a realistic dense urban distribution of gNB s. We consider that 1500 user are active at the same time with multi-megabit traffic demands, which is a realistic number for a single operator in a mid-size city like Leganés. Furthermore, in this scenario we configure the network using operator-calibrated parameters and classify users into three traffic-demand tiers, reflecting real-world application profiles:

- Tier 1: 60% of users, using messaging, email, web browsing, and GPS navigation apps ($\sim 1 \text{ Mbps}$ downlink).
- Tier 2: 30% of users, engaged in social media, video calls, or light gaming ($\sim 5 \text{ Mbps}$ downlink).
- Tier 3: 10% of users, running high-demand services like HD/4K streaming, cloud backups, and tethering ($\sim 25 \text{ Mbps}$ downlink).

Hence, each tier is assigned a weight corresponding to its

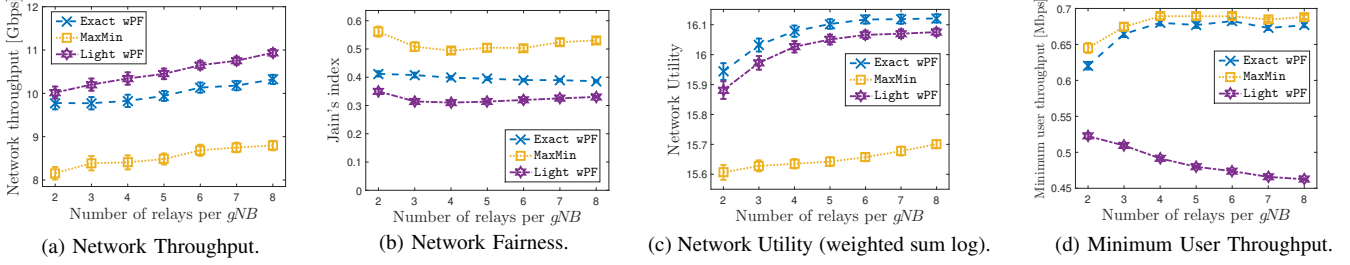


Figure 3: Wireless relay network with 10 gNB s and $U = 1500$ users. Scenario: REALISTIC.

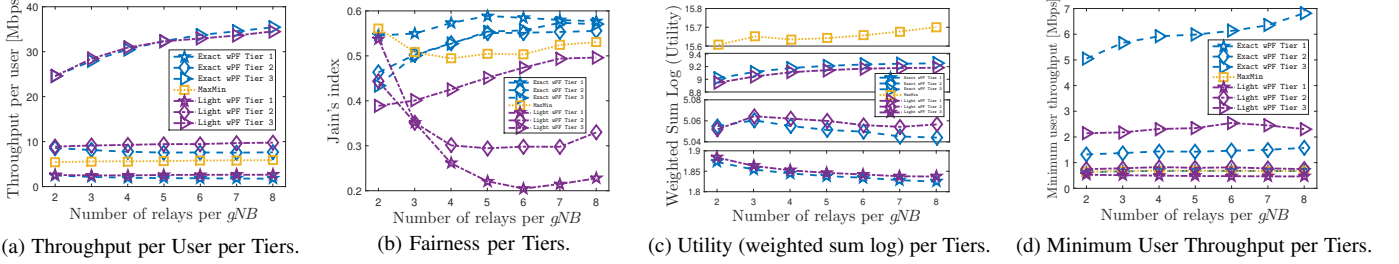


Figure 4: Wireless relay network with 10 gNB s and $U = 1500$ users. Scenario: REALISTIC.

traffic demand: 1, 5 and 25 for Tiers 1, 2, and 3 respectively. This weighted model allows us to investigate the real impact of prioritizing resources based on actual user needs, aligned with industry and research data [17].

Figure 3 summarizes network-wide performance, while Figure 4 disaggregates results by tier. In Figure 3a, Exact wPF achieves around 20% higher total throughput than MaxMin, confirming that resource prioritization aligned with demand yields significant efficiency gains.

If we zoom into the results of our exact solution and the associated simplified heuristic, Figure 4a clearly shows differentiated throughput across tiers:

- Tier 1 users reach ~ 1 – 2 Mbps with either Exact wPF or Light wPF—sufficient for their application needs.
- Tier 2 users reach ~ 7 – 9 Mbps with Exact wPF and up to ~ 10 Mbps with Light wPF—outperforming MaxMin despite higher demands of those users.
- Tier 3 users benefit the most, with high-throughput allocations that reflect their weight and need, and with little difference between Exact wPF and Light wPF.

The rate-per-tier scales proportionally to the assigned weights, which aligns well with user requirements.

Figures 3b and 4b report fairness. While MaxMin excels in total fairness (by design), Exact wPF achieves better intra-tier fairness—each group receives fair treatment relative to its role in the network. This validates that assigning differentiated priorities enables tailored and equitable resource distribution. However, Light wPF oversimplifies network constraints and, as a consequence, is not able to handle fairness correctly. Indeed, it achieves the worst level of fairness performance, which becomes very low especially for Tiers 1 and 2.

Figures 3c and 4c report network utility, defined as the sum of the weighted logarithms of user rates, which serves as a well-established metric to capture the trade-off between

fairness and total capacity. As shown in Figure 3c, the Exact wPF scheme achieves a significantly better balance between these two competing objectives with respect to MaxMin, even in realistic topologies. The utility of Light wPF is comparable to the one of Exact wPF, yet inferior. Figure 4c further breaks down the utility by user tiers, confirming that the aggregate utility is effectively distributed according to user demand with both Exact wPF and Light wPF. Notably, the increasing utility trend observed in Tier 3 highlights that the Exact wPF scheduler behaves as intended: it successfully prioritizes users with higher weights and demand, even if this results in a slight reduction in overall network fairness, as reflected earlier in Figure 3b. While Light wPF manages to achieve the same utility trend, it is worth noting that it imposes much worse fairness operations due to its sub-optimality.

Finally, Figures 3d and 4d show that even the minimum user rate achieved with Exact wPF is as good as with MaxMin and aligned with service expectations per tier, although Light wPF performs significantly worse than the other schemes. With Exact wPF, Tier 1 users achieve a minimum rate close to their requirements, while Tiers 2 and 3 are also served adequately. By contrast, MaxMin may boost the absolute minimum, but without guaranteeing that high-demand users are adequately served.

Complexity comparison. We have estimated that the execution of Exact wPF for instances of the Realistic scenario can vary from 40 000 to 60 000 operations in MATLAB, depending on the number of relays employed, while Solver wPF required about 3.4 million operations to achieve the same results of Exact wPF. The contrast is striking and points out the importance of our analysis when it comes to real implementation. Besides, we estimated that Light wPF allowed us to further reduce the number of operations required by a factor of 3 or 4 with respect to Exact wPF, i.e., down

to about 12 000 operations per topology realization, although with very poor fairness results, which might not be acceptable. Overall, this comparison illustrates how complex legacy solvers become soon unusable for medium large topologies, while optimal solutions can be attained at very low cost if one exploits the analytical structure of weighted proportional fair resource allocation in wireless relay networks.

VII. CONCLUSIONS

In this paper we have described how to tackle the resource allocation problem in wireless relay networks such as the ones enabled by 3GPP/ETSI architectures for 5G networks and beyond. Specifically, we considered the challenge of achieving weighted proportional fairness among users. To this end, we have analyzed the resource and throughput distribution scenario in detail, accounting for backhaul constraints and realistic signal propagation in dense urban environment with low power relays distributed on a 3D field (nonetheless, different wireless propagation and relay positioning scenarios can be easily used in the analysis). Hence, we were able to formulate an optimization problem whose convex log-shaped structure allows for the design of smart and efficient optimization algorithms.

Our proposed solution, `Exact wPF` is exact, i.e., it achieves the optimum network utility, and runs in linear time, which is an outstanding improvement with respect to using legacy convex solvers that run in cubic time.

We have shown how `Exact wPF` allows for balancing between throughput and fairness, hence highlighting the importance of disposing of a tool for fast weighted proportional fairness allocation of network resources. We have also shown that further simplified approaches fail in guaranteeing decent fairness performance.

Noteworthy, `Exact wPF` can be used with any topological distribution of nodes and, in particular, it is straightforward to use it for the optimization of realistic and large environments, as we have shown in the paper.

As future work, we plan to extend our framework in several directions. First, we aim to incorporate the generalized α -fairness metric, which allows tuning the balance between efficiency and fairness beyond the logarithmic or max-min case, potentially accommodating different operator objectives. Hence, we intend to study capacity-maximizing variants of our algorithm to better support traffic engineering strategies focused on maximizing aggregate throughput. Finally, we will explore the integration of our scheme into a fully discrete resource scheduling model that accounts for the quantization of bandwidth into Physical Resource Blocks (PRBs), thereby enabling practical deployment in 5G and beyond systems with standard-compliant scheduling constraints.

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