

Faulty yet Secure: Sum Secrecy Rate Optimization under Faulty Reconfigurable Intelligent Surfaces

Nairy Moghadas Gholian, Placido Mursia, Matthias Hollick, Arash Asadi

Abstract—Hardware faults in reconfigurable intelligent surfaces (RISs) disrupt their phase alignment and degrade control over the wireless channel, exposing systems to new physical layer security (PLS) vulnerabilities. Faulty reflecting elements, often triggered by manufacturing defects, aging, or environmental stress, distort beamforming, causing signal leakage that can be exploited by eavesdroppers. Despite this risk, existing studies have neither quantified the effect of such faults on the sum secrecy rate (SSR) nor developed mitigation strategies. To address this gap, we propose a solution (i.e., FIERSE) which jointly optimizes the base station (BS) precoder and RIS phase shifters to maximize the SSR under partial hardware faults. To further ensure fairness among user equipments (UEs), we exploit the leakage-aware property of the signal-to-leakage-and-noise ratio (SLNR) formulation and develop two max-min SLNR schemes: one based on perfect channel state information (CSI) assumption and the other on statistical CSI. Simulation results show that FIERSE preserves strong secrecy performance even with 86% faulty elements, achieving nearly 92% higher SSR than the fault-unaware baseline. The max-min SLNR schemes achieve approximately 32% of FIERSE's secrecy performance while providing high fairness (Jain's index ≈ 0.9) under about 62% faulty elements. Our findings demonstrate that fault-aware design is crucial for maintaining secrecy in RIS-assisted networks undergoing partial faults. Depending on system requirements, FIERSE offers higher secrecy efficiency, whereas max-min SLNR schemes provide improved fairness and robustness.

Index Terms—Reconfigurable intelligent surfaces, faulty antennas, physical layer security, fairness.

I. INTRODUCTION

Reconfigurable intelligent surfaces (RISs) are emerging enablers for 6th generation (6G) networks, improving data rate, network coverage, localization, sensing, and more recently, physical layer security (PLS) [1]–[5]. RISs can manipulate the characteristics of the reflected signal (most commonly phase) to direct/focus resulting transmission in the desired direction. Although RIS technology is still in the early stages of practical deployment, recent experimental prototypes and testbed implementations have demonstrated the feasibility of programmable wireless environments and increased significant research interest [6].

RIS for PLS. The large aperture of RISs allows for highly directive beamforming, making them an attractive solution for enhancing PLS [7], in particular for *reducing the signal leakages and eavesdropping*. As secure connectivity is a

key 6G requirement due to massive connectivity and open programmable radio environments, RIS-enabled PLS has received growing attention. Authors in [8] demonstrated that the communication secrecy rate can be significantly improved by leveraging the additional degree of freedom provided by RISs to reduce the channel quality and decoding capability of the eavesdropper. In the mentioned work, the basic assumption is that the RIS works perfectly as expected.

RISs are prone to errors/faults. The majority of the prior works have analyzed RIS-assisted networks under transmitter and receiver hardware impairments, such as phase noise, power amplifier non-linearities, and I/Q (in-phase/quadrature) imbalance [9]–[11], while assuming the RIS operates ideally. However, in practice, the RIS itself is exposed to hardware impairments.

Impact of RIS faultiness in PLS. Previous studies have examined secrecy rate optimization under *inherent hardware impairments* of RIS elements, including phase noise, quantization errors, amplification noise, and residual distortion [12]–[16]. These models assume that all RIS elements remain functional but subject to a random added phase error. However, in practical deployments, particularly in outdoor environments, RIS panels can suffer from partial damage or permanent malfunction due to heat, humidity, or physical impact. Over time, such effects can accumulate, leading to a subset of elements becoming completely uncontrollable. Because maintenance or replacement of large outdoor RIS installations (e.g., facade-mounted panels) is costly and often impractical, fault-tolerant design becomes essential. Unlike the conventional hardware-impairment models, which treat each RIS element as functional but noisy, our *faulty-RIS* model considers a more severe scenario where impaired elements lose deterministic amplitude and phase control altogether and behave as random scatterers [17], [18]. This setting is also fundamentally different from imperfect channel state information (CSI) models, where channel estimation errors are considered but the RIS elements remain controllable. In contrast, faulty elements represent hardware failures that introduce uncontrollable reflection responses. Furthermore, faulty RIS elements disrupt the array structure, breaking the regular lattice and leading to distorted beam patterns. Unlike CSI errors, which are typically modeled as additive perturbations, hardware faults affect the signal multiplicatively by altering its propagation through the RIS, resulting in fundamentally different impacts.

This paper introduces FIERSE (fault-resistant intelligently-empowered RIS with security enhancements), a fault-aware optimization framework that maximizes the sum secrecy rate (SSR) across legitimate user equipments (UEs) under RIS element faults. The design combines alternating optimization (AO), quadratic transform, semi-definite program-

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minimization (SDP), majorization-minimization (MM)/successive convex approximation (SCA) procedure (in this work, MM step corresponds to a minorization–maximization update), followed by rank-1 extraction. To characterize the performance–fairness–complexity trade-off, we also develop max–min signal-to-leakage-and-noise ratio (SLNR) schemes for both perfect and statistical CSI, targeting worst-UE leakage suppression [19]. The results demonstrate that fault awareness is essential, and ignoring RIS faults leaves sizable secrecy losses that persist even with large transmit power or aperture.

A. Contributions

The main contributions of this paper are summarized as follows:

- **First SSR optimization framework for faulty RIS-assisted networks:** We investigate a downlink multi-user (MU) multiple-input single-output (MISO) system aided by a RIS in which a subset of reflecting elements suffer from hardware faults and act as uncontrolled scatterers. Unlike prior works that model RIS impairments as small added phase errors, we explicitly capture random amplitude and phase responses at faulty elements. This modeling reveals that neglecting such faults not only wastes useful signal power but also creates strong side-lobes that severely compromise PLS. To the best of the authors’ knowledge, this is the first work to address SSR degradation in the presence of a realistic faulty RIS.
- **Problem formulation for robust secrecy optimization:** To mitigate the leakage introduced by faulty elements, we jointly optimize the base station (BS) precoder and the remaining functional RIS phase shifters with the objective of enhancing SSR. Specifically, we propose the FIERSE algorithm, which maximizes the SSR under practical unit-modulus and transmit power constraints. In addition, to capture fairness–complexity trade-offs, we design two complementary max–min SLNR schemes under both perfect and statistical CSI. These formulations directly account for the detrimental effect of element faults on signal leakage, a dimension overlooked in existing RIS-aided PLS studies.
- **Problem solution and algorithmic design:** The resulting optimization problems are highly nonconvex due to the coupling between RIS phase shifters and BS precoder, together with the fractional secrecy objective. We develop an AO framework that integrates quadratic transform, semi-definite relaxation (SDR), and MM-based SCA. In particular, we derive new MM-based SCA surrogates for the faulty-element SSR objective, which requires separate concave minorizations for the precoder and the phase shifter, leading to a novel AO structure. For the SLNR-based schemes, we reformulate the max–min problems into convex feasibility subproblems and solve them efficiently using bisection and SDP.
- **Performance evaluation and insights:** Compared to baseline schemes, our proposed algorithms substantially reduce information leakage and enhance secrecy. In particular, FIERSE retains about 25% of the upper-bound SSR value even when roughly 86% of the RIS elements are faulty, outperforming the baseline scheme, which neglects the existence of faults. The SLNR-based variants preserve acceptable secrecy performance while guaranteeing UE fairness,

with Jain’s index values above 0.9 and reducing computational complexity. These results highlight the penalty of ignoring faulty elements and confirm an inherent trade-off between secrecy and fairness in the presence of faulty RIS.

B. Related work

Our proposed algorithms aim to provide robust PLS in the presence of faulty RIS, which overlaps with two streams of research: fault-aware modeling of RIS and PLS optimization with RIS assistance. We will review each category separately.

Fault-aware RIS modeling: Diagnostics for faulty phased arrays, which are conceptually related to RISs, often rely on compressed sensing with probe measurements to recover patterns and detect anomalies [20], [21]. For RIS, the literature is relatively limited and mostly diagnostic. These studies aim to detect faulty elements within RIS and assess their impact in terms of signal attenuation and phase shift. Methods have been proposed to detect irregularly behaving reflecting elements even under limited or absent CSI [22]. Other studies develop diagnostic strategies that jointly identify faulty elements and facilitate UE localization [23]. Similarly, [24] suggests an algorithm for diagnosing faulty elements in RIS using compressed sensing. Fault detection has additionally been formulated as an equivalent channel estimation problem, where dedicated pilot signals are used to determine the condition of each element [25]. These works indicate that faulty RIS elements can be identified and their reflection characteristics (e.g., amplitude attenuation and phase response) can be estimated through dedicated pilot-based estimation and diagnostic procedures in literature. In particular, since fault detection can be formulated as an equivalent channel estimation problem, the effective cascaded BS-RIS-UE channel in the presence of faulty elements can be estimated during training. In addition to detection and estimation approaches, recent work has also investigated the analytical impact of faulty RIS elements. In particular, the misspecified Cramér–Rao bound (MCRB) has been used to model and quantify the sensing and signal-to-interference-plus-noise ratio (SINR) degradation induced by faulty RIS elements [18].

Optimizing PLS with RIS: Recent studies show that RISs can improve secrecy rate. In particular, [26] demonstrates secrecy gains in a single-antenna setting (vs. no-RIS baseline) even under an adversarial eavesdropper. Classical secure designs inject artificial noise (AN) to jam eavesdroppers. For example, [27] jointly optimizes precoder, RIS phase shifters, and a jamming signal via iterative Block Coordinate Descent (BCD) with Lagrangian duality/MM for a single-UE, single-eavesdropper setup. While effective, AN raises power consumption and typically requires more transmit antennas than the eavesdropper [28]. Moreover, [29] maximizes secrecy rate by jointly optimizing the BS precoder and RIS phase shifters via two distinct algorithms, namely element-wise BCD and AO-MM. Furthermore, authors in [16] study secure MU transmission with hardware-impaired BS, RIS, and UEs, where RIS imperfections are modeled as additive phase-noise distortions on fully functional elements. The authors maximize the weighted minimum approximate ergodic secrecy rate (WMAESR) to ensure robustness under stochastic hardware uncertainty.

To the best of our knowledge, *sum secrecy rate maximization with completely non-functional RIS elements* and a fault-aware co-design have not been addressed. Therefore, in this paper, we propose the FIERSE framework and max-min SLNR designs to fill the mentioned research gap.

The remainder of the paper is structured as follows: Section I-B discusses the related work, Section II details the system and fault model, Section III formulates the optimization problems, derives the algorithms, and analyzes their complexity. Section IV presents the simulation results, and finally Section V concludes the paper.

Notation: Matrices and vectors are denoted in uppercase and lowercase bold font, respectively. $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, $\text{tr}(\cdot)$, and $\text{Rank}(\cdot)$ stand for transposition, complex conjugation, Hermitian transposition, trace, and rank of a square matrix, respectively. Additionally, $|\cdot|$ denotes the absolute value of a complex number. Moreover, $i = \sqrt{-1}$ is the imaginary number, $\text{diag}(\mathbf{x})$ denotes a diagonal matrix whose diagonal entries are given by vector $\mathbf{x}$, and $\text{diag}(\mathbf{A})$ denotes the vector formed by the diagonal elements of square matrix $\mathbf{A}$. $\Re\{\cdot\}$ stands for the real part of a complex number and $[\mathbf{a}]_m$ represents the m -th entry of the vector $\mathbf{a}$. Finally, $\mathcal{O}$ stands for the big-O notation and $\{\cdot\} \succeq 0$ indicates a positive semi-definite (psd) matrix. An asterisk *superscript without parentheses* (e.g. $\mathbf{x}^*$) designates the final optimized value of a variable. Angled brackets represent the Frobenius inner product, $(\mathbf{A}, \mathbf{B}) \triangleq \text{Tr}(\mathbf{A}^H \mathbf{B})$ which reduces to the ordinary dot product when $\mathbf{A}$ and $\mathbf{B}$ are real.

II. SYSTEM AND FAULT MODEL

We consider a downlink MU MISO network employing a RIS, as depicted in Fig. 1. The RIS suffers from faulty elements that create unintended signal leakage, thereby degrading secrecy performance in the presence of an eavesdropper. The goal is to enhance the performance of legitimate UEs while mitigating this fault-induced secrecy degradation. We consider a BS equipped with M antennas serving K legitimate UEs where the direct links between the BS and each UE, as well as the eavesdropper, are assumed to be blocked by obstacles. Consequently, both legitimate UEs and the eavesdropper receive the BS signal only via the RIS. We model the RIS as a uniform planar array (UPA) on the xz -plane with N_x and N_z elements along the x - and z -axes, respectively, which results in a total of $N = N_x N_z$ elements. Each element is equipped with an independently tunable phase shifter that imposes a controllable phase $\theta_n \in [0, 2\pi)$, yielding the reflection matrix $\Phi = \text{diag}([e^{j\theta_1}, \dots, e^{j\theta_N}]) \in \mathbb{C}^{N \times N}$. All legitimate UEs and the eavesdropper employ single-antenna receivers. Accordingly, the received baseband signal at the k -th UE is given by

$$\begin{aligned} y_k &= \mathbf{h}_k^H \Phi \mathbf{G} \mathbf{W} \mathbf{s} + n_k \\ &= \mathbf{h}_k^H \Phi \mathbf{G} \mathbf{w}_k s_k + \sum_{j \neq k} \mathbf{h}_k^H \Phi \mathbf{G} \mathbf{w}_j s_j + n_k \in \mathbb{C}, \forall k \in \mathcal{K} \end{aligned} \quad (1)$$

where $\mathcal{K} = \{1, \dots, K\}$, $\mathbf{G} \in \mathbb{C}^{N \times M}$, and $\mathbf{h}_k \in \mathbb{C}^{N \times 1}$, correspond to the set of legitimate UEs, BS-RIS channel, and RIS-UE channel, respectively. The transmit precoder at the BS is defined by $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_K] \in \mathbb{C}^{M \times K}$ where $\mathbf{w}_k \in \mathbb{C}^{M \times 1}$ is the precoding vector that directs the transmitted signal

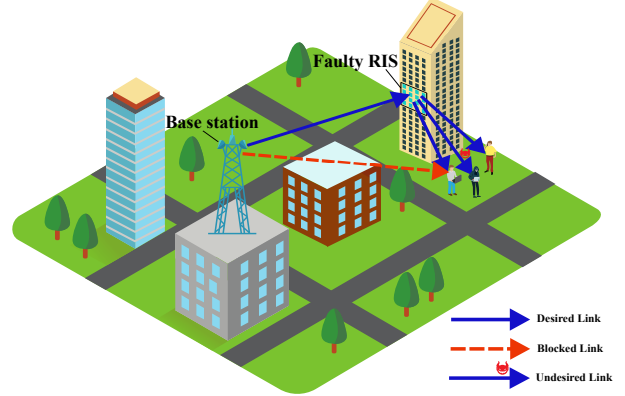


Fig. 1: Communication network in the presence of faulty RIS

through the effective channel between the BS and the k -th legitimate UE through the RIS, with $\sum_{k \in \mathcal{K}} \|\mathbf{w}_k\|^2 \leq P$ where P stands for the maximum transmit power budget. Furthermore, $\mathbf{s} = [s_1, \dots, s_K]^T \in \mathbb{C}^{K \times 1}$ denotes the transmit symbol vector, with $\mathbb{E}[|s_k|^2] = 1$, $\forall k \in \mathcal{K}$ and s_k is the transmitted data for the k -th legitimate UE. $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ denotes the additive white Gaussian noise (AWGN) at the k -th legitimate UE. Consequently, we rewrite (1) as

$$y_k = \mathbf{v}^H \bar{\mathbf{H}}_k \mathbf{w}_k s_k + \sum_{j \neq k} \mathbf{v}^H \bar{\mathbf{H}}_k \mathbf{w}_j s_j + n_k, \quad (2)$$

where $\mathbf{v} = \text{diag}(\Phi^H) \in \mathbb{C}^{N \times 1}$ and $\bar{\mathbf{H}}_k = \text{diag}(\mathbf{h}_k^H) \mathbf{G} \in \mathbb{C}^{N \times M}$. In an ideal scenario where all the RIS elements are fully functioning, the SINR at legitimate UE k can be expressed as

$$\text{SINR}_k = \frac{|\mathbf{v}^H \bar{\mathbf{H}}_k \mathbf{w}_k|^2}{\sum_{j \neq k} |\mathbf{v}^H \bar{\mathbf{H}}_k \mathbf{w}_j|^2 + \sigma_k^2}. \quad (3)$$

Similarly, we define the received signal at the eavesdropper as

$$y_e = \mathbf{v}^H \bar{\mathbf{H}}_e \mathbf{w}_k s_k + \sum_{j \neq k} \mathbf{v}^H \bar{\mathbf{H}}_e \mathbf{w}_j s_j + n_e, \quad (4)$$

where $\bar{\mathbf{H}}_e = \text{diag}(\mathbf{h}_e^H) \mathbf{G} \in \mathbb{C}^{N \times M}$ is the cascaded channel from the BS to eavesdropper, via the RIS, $\mathbf{h}_e \in \mathbb{C}^{N \times 1}$ is the RIS-eavesdropper channel and $n_e \sim \mathcal{CN}(0, \sigma_e^2)$ is the AWGN at the eavesdropper. Similar to (3), when all RIS elements are functioning, the SINR at the eavesdropper e is given by

$$\text{SINR}_e = \frac{|\mathbf{v}^H \bar{\mathbf{H}}_e \mathbf{w}_k|^2}{\sum_{j \neq k} |\mathbf{v}^H \bar{\mathbf{H}}_e \mathbf{w}_j|^2 + \sigma_e^2}. \quad (5)$$

Remark 1: We assume that the eavesdropper has perfect decoding capabilities and can therefore simultaneously intercept all transmitted signals [30]. This modeling choice follows the commonly adopted worst-case adversary assumption in PLS studies, where the eavesdropper is considered to have perfect knowledge of its channel and strong decoding capability [31], [32].

A. Faulty element model

To model the behavior of faulty RIS, we assume that each faulty element exhibits an uncontrollable reflection coefficient modeled as a complex-valued random variable with uniformly

distributed amplitude and phase. Let $B \leq N$ denote the number of faulty elements and let $\bar{N} = N - B$ denote the number of functional elements. Accordingly, we express the RIS vector as $\mathbf{v} = [\mathbf{v}_R^T \mathbf{v}_B^T]^T$ and the overall channel as $\bar{\mathbf{H}}_i = [\bar{\mathbf{H}}_{R,i}^T \bar{\mathbf{H}}_{B,i}^T]^T$, $\forall i \in \{k, e\}$, where each faulty entry $v_{B,b}$, $b = 1, \dots, B$ is a random variable (RV) satisfying $|v_{B,b}| \sim \mathcal{U}(0, 1)$ and $\angle v_{B,b} \sim \mathcal{U}[0, 2\pi)$. Note that the locations and reflection coefficients of faulty elements are assumed to be known. In practice, such information can be obtained through RIS diagnostic and fault detection techniques proposed in the literature [20], [22], [24], [33], which identify malfunctioning elements and characterize their reflection responses. To model the randomness of faulty RIS elements, we assume that, for each channel realization, the reflection coefficient $v_{B,b}$ of every faulty element is randomly drawn once and remains fixed throughout the transmission interval (e.g., within a channel coherence block). This randomness does not represent fast time-varying fluctuations during symbol transmission. Instead, it reflects a practical scenario in which faulty RIS elements behave as uncontrollable scatterers with fixed but unknown amplitude and phase responses during a given transmission period. We remark that this uniform fault model presents a simplified worst-case abstraction of completely uncontrollable RIS elements. In the considered faulty RIS model, the sub-vector $\mathbf{v}_R \in \mathbb{C}^{\bar{N} \times 1}$ represents the tunable functional elements, whereas $\mathbf{v}_B \in \mathbb{C}^{B \times 1}$ denotes the uncontrollable faulty elements. The channel associated with the functional elements is represented by $\bar{\mathbf{H}}_{R,i} \in \mathbb{C}^{\bar{N} \times M}$, while $\bar{\mathbf{H}}_{B,i} \in \mathbb{C}^{B \times M}$ corresponds to the channel of the faulty elements for each $i \in \{k, e\}$. Accordingly, we reformulate the received signal in (2) as follows

$$\begin{aligned} y_i &= [\mathbf{v}_R^H \mathbf{v}_B^H] \begin{bmatrix} \bar{\mathbf{H}}_{R,i} \\ \bar{\mathbf{H}}_{B,i} \end{bmatrix} \mathbf{w}_k s_k + \sum_{j \neq k} [\mathbf{v}_R^H \mathbf{v}_B^H] \begin{bmatrix} \bar{\mathbf{H}}_{R,i} \\ \bar{\mathbf{H}}_{B,i} \end{bmatrix} \mathbf{w}_j s_j + n_i \\ &= (\mathbf{v}_R^H \bar{\mathbf{H}}_{R,i} + \mathbf{h}_{Bd,i}^H) \mathbf{w}_k s_k + \sum_{j \neq k} (\mathbf{v}_R^H \bar{\mathbf{H}}_{R,i} + \mathbf{h}_{Bd,i}^H) \mathbf{w}_j s_j + n_i, \end{aligned} \quad (6)$$

where $\mathbf{h}_{Bd,i} = \mathbf{v}_B^H \bar{\mathbf{H}}_{B,i} \in \mathbb{C}^{M \times 1}$, $\forall i \in \{k, e\}$ models the distortion caused by the faulty RIS elements and must not be confused with the direct Line-of-Sight (LoS) channel between the BS and the UE. Using (6), the SINR observed at the legitimate UE k is therefore given by

$$\text{SINR}_k = \frac{|(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} + \mathbf{h}_{Bd,k}^H) \mathbf{w}_k|^2}{\sum_{j \neq k} |(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} + \mathbf{h}_{Bd,k}^H) \mathbf{w}_j|^2 + \sigma_k^2}. \quad (7)$$

Similarly, the SINR experienced by the eavesdropper e is given by

$$\text{SINR}_e = \frac{|(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,e} + \mathbf{h}_{Bd,e}^H) \mathbf{w}_k|^2}{\sum_{j \neq k} |(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,e} + \mathbf{h}_{Bd,e}^H) \mathbf{w}_j|^2 + \sigma_e^2}. \quad (8)$$

Next, we adopt SLNR as a design metric, which quantifies the ratio between the intended signal power and the leakage toward the eavesdropper [19]. Unlike SINR, SLNR directly penalizes signal leakage toward unintended receivers and therefore is particularly suitable for secure designs. Under this definition, the SLNR for UE k is given by

$$\text{SLNR}_k = \frac{|(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} + \mathbf{h}_{Bd,k}^H) \mathbf{w}_k|^2}{|(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,e} + \mathbf{h}_{Bd,e}^H) \mathbf{w}_k|^2 + \sigma_e^2}. \quad (9)$$

B. Secrecy rate definition

We first recall the achievable information rates of the legitimate UE k and the eavesdropper e . By Shannon's capacity formula and the SINR expressions in (3) and (5), these rates are given by $R_k = \log_2(1 + \text{SINR}_k)$ and $R_e = \log_2(1 + \text{SINR}_e)$, respectively [34]. Thus, the secrecy rate is defined as the positive part of their difference [31], as follows

$$\begin{aligned} R_{s,k} &\triangleq [R_k - R_e]^+ = \\ &\left[\log_2 \left(1 + \frac{|(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} + \mathbf{h}_{Bd,k}^H) \mathbf{w}_k|^2}{\sum_{j \neq k} |(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} + \mathbf{h}_{Bd,k}^H) \mathbf{w}_j|^2 + \sigma_k^2} \right) \right. \\ &\quad \left. - \log_2 \left(1 + \frac{|(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,e} + \mathbf{h}_{Bd,e}^H) \mathbf{w}_k|^2}{\sum_{j \neq k} |(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,e} + \mathbf{h}_{Bd,e}^H) \mathbf{w}_j|^2 + \sigma_e^2} \right) \right]^+. \end{aligned} \quad (10)$$

where $[x]^+ \triangleq \max\{x, 0\}$ ensures the result is nonnegative, and all the rates are reported in b/s/Hz. Collecting per-UE secrecy rates, we define the sum secrecy rate (SSR) as $\text{SSR} = \sum_{k \in \mathcal{K}} R_{s,k}$. The SSR measures the aggregate confidential throughput delivered to all legitimate UEs in the presence of the eavesdropper and serves as a standard system-level benchmark. In the next section, we use SSR as the main objective and formulate the corresponding optimization problem under the transmit-power and unit-modulus constraints.

III. PROBLEM FORMULATION AND ALGORITHM DESIGN

In this section, we formulate the overall SSR maximization problem and outline its solution, as detailed in Section III-A. Specifically, we adopt AO to handle the variable coupling, alternating between the resulting subproblems until convergence. Subsequently, we introduce the max-min SLNR optimization problem in Section III-B, which maximizes the minimum SLNR among several UEs. Our proposed method mitigates the leakage and strengthens the security of the intended UEs.

A. Sum secrecy rate maximization

The optimization problem for jointly optimizing precoder $\mathbf{w}_k$ and functioning RIS phase shifter $\mathbf{v}_R$ is formulated as

$$\begin{aligned} &\underset{\mathbf{w}_k, \mathbf{v}_R}{\text{maximize}} && \sum_{k \in \mathcal{K}} R_{s,k} \\ &\text{subject to} && \text{C1: } \sum_{k \in \mathcal{K}} \|\mathbf{w}_k\|^2 \leq P, \\ &&& \text{C2: } |v_{R,n}| = 1, n = 1, \dots, \bar{N}. \end{aligned} \quad (11)$$

where C2 enforces unit amplitude for each RIS element while permitting phase shifts over $[0, 2\pi)$. This unit-modulus constraint is inherently nonconvex. In fact, even when the beamforming vectors are fixed, the resulting phase-only optimization reduces to a constant-modulus quadratic program, which has been shown to be NP-hard [35]. Here, the difficulty is compounded because $R_{s,k}$ is the difference of two logarithms, making the objective itself nonconcave, and each SINR term couples $\mathbf{v}_R$ and $\mathbf{w}_k$ multiplicatively. Consequently, the overall problem (11) is jointly nonconvex in $\mathbf{v}_R$ and $\mathbf{w}_k$. Therefore, a single convex program cannot solve it. In the sequel, we adopt the AO strategy to optimize the BS precoder vector of the legitimate UE k via quadratic transform, SDP,

and MM-based SCA approaches for a fixed RIS phase shifter. Later, with BS precoder vector of k -th UE held constant, we optimize the phase shift vector $\mathbf{v}_R$ of the RIS. Repeating these two convex subproblems yields a monotonically non-decreasing value of the MM-based SCA surrogate of the SSR objective. As shown later, standard MM theory ensures that each MM-based SCA block update converges to a stationary point of its SDR-relaxed subproblem.

Optimizing BS precoder: Let the RIS phase shift vector $\mathbf{v}_R$ be fixed. The outer problem (11) then reduces to maximizing the SSR with respect to the downlink precoder vector $\{\mathbf{w}_k\}_{k \in \mathcal{K}}$. First, we define the psd rank-1 matrix for the precoder as $\mathbf{W}_k \triangleq \mathbf{w}_k \mathbf{w}_k^H \succeq \mathbf{0}$. Moreover, we introduce $\mathbf{Q}_k$, $\forall k \in \mathcal{K}$ and $\mathbf{Z}_e$ as follows

$$\mathbf{Q}_k \triangleq (\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} + \mathbf{h}_{Bd,k}^H)^H (\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} + \mathbf{h}_{Bd,k}^H), \quad (12)$$

$$\mathbf{Z}_e \triangleq (\mathbf{v}_R^H \bar{\mathbf{H}}_{R,e} + \mathbf{h}_{Bd,e}^H)^H (\mathbf{v}_R^H \bar{\mathbf{H}}_{R,e} + \mathbf{h}_{Bd,e}^H), \quad (13)$$

The legitimate UE and eavesdropper SINRs are

$$\begin{aligned} \gamma_k(\mathbf{W}_k) &= \frac{\text{Tr}(\mathbf{Q}_k \mathbf{W}_k)}{\sum_{j \neq k} \text{Tr}(\mathbf{Q}_k \mathbf{W}_j) + \sigma_k^2}, \\ \gamma_e(\mathbf{W}_k) &= \frac{\text{Tr}(\mathbf{Z}_e \mathbf{W}_k)}{\sum_{j \neq k} \text{Tr}(\mathbf{Z}_e \mathbf{W}_j) + \sigma_e^2}. \end{aligned} \quad (14)$$

Consequently, according to (10), the per-UE secrecy rate is defined as $R_{s,k} = \log_2(1 + \gamma_k(\mathbf{W}_k)) - \log_2(1 + \gamma_e(\mathbf{W}_k))$. For tractability, we set

$$\begin{aligned} S_k &= \text{Tr}(\mathbf{Q}_k \mathbf{W}_k), & I_k &= \sum_{j \neq k} \text{Tr}(\mathbf{Q}_k \mathbf{W}_j) + \sigma_k^2, \\ L_k &= \text{Tr}(\mathbf{Z}_e \mathbf{W}_k), & E_k &= \sum_{j \neq k} \text{Tr}(\mathbf{Z}_e \mathbf{W}_j) + \sigma_e^2. \end{aligned} \quad (15)$$

By applying the change-of-base formula $\log_2(x) = \frac{\ln x}{\ln 2}$ and substituting $\gamma_k = \frac{S_k}{I_k}$ and $\gamma_e = \frac{L_k}{E_k}$ from (15) into (10), the SSR expression yields

$$\sum_{k \in \mathcal{K}} R_{s,k}(\mathbf{W}_k) = \frac{1}{\ln 2} \sum_{k \in \mathcal{K}} \underbrace{\left[\ln\left(\frac{S_k + I_k}{I_k}\right) - \ln\left(\frac{L_k + E_k}{E_k}\right) \right]}_{\triangleq \Delta_k}. \quad (16)$$

where the constant factor $\frac{1}{\ln 2}$ is omitted in the sequel since it does not affect the problem.

Remark. Every denominator in (16) is strictly positive since $I_k \geq \sigma_k^2 > 0$ and $E_k \geq \sigma_e^2 > 0$. Thus, the numerators are nonnegative, hence Δ_k is well-defined [36].

Lemma 1. We initialize an arbitrary but finite tuple of precoder matrices $\mathbf{W}_k$, $\forall k \in \mathcal{K}$. For that $\mathbf{W}_k$, let S_k, I_k, L_k, E_k be defined as in (15). Thus, Δ_k can be written as

$$\begin{aligned} \Delta_k &= \ln(2\rho_k \sqrt{S_k + I_k} - \rho_k^2 I_k) \\ &\quad - \ln(2\mu_k \sqrt{L_k + E_k} - \mu_k^2 E_k). \end{aligned} \quad (17)$$

achieved at $\rho_k = \sqrt{(S_k + I_k)/I_k}$ and $\mu_k = \sqrt{(L_k + E_k)/E_k}$.

Proof: Applying Corollary 2 of the quadratic-transform framework in [36] to $f(x, y) = \ln((x + y)/y)$ twice, with $(x, y) = (S_k, I_k)$ and (L_k, E_k) , and subtracting the resulting concave envelopes yields (17). ■

Corollary 1. Let $\rho = \{\rho_k\}_{k \in \mathcal{K}}$, $\mu = \{\mu_k\}_{k \in \mathcal{K}}$, and $\mathcal{W} \triangleq \{\mathbf{W}_k\}_{k \in \mathcal{K}}$. For any fixed ρ, μ , define

$$\Psi(\mathcal{W}; \rho, \mu) \triangleq \sum_{k \in \mathcal{K}} \left[2\rho_k \sqrt{S_k + I_k} - \rho_k^2 I_k - 2\mu_k \sqrt{L_k + E_k} + \mu_k^2 E_k \right]. \quad (18)$$

where $\Psi(\mathcal{W}; \rho, \mu)$ is a tractable surrogate objective of $\sum_{k \in \mathcal{K}} R_{s,k}(\mathbf{W}_k)$ which is obtained from the quadratic-transform representation of the legitimate UEs' and eavesdropper's SINR terms.

Let $\Psi(\mathcal{W}) \triangleq \sum_{k \in \mathcal{K}} [f_k(\mathbf{W}_k) - g_k(\mathbf{W}_k)]$ where the constituent functions are defined as follows

$$\begin{aligned} f_k(\mathbf{W}_k) &= 2\rho_k \sqrt{S_k + I_k} - \rho_k^2 I_k, \\ g_k(\mathbf{W}_k) &= 2\mu_k \sqrt{L_k + E_k} - \mu_k^2 E_k. \end{aligned} \quad (19)$$

and $f_k(\mathbf{W}_k)$ and $g_k(\mathbf{W}_k)$ are inherently concave. Since $g_k(\mathbf{W}_k)$ is concave, its first-order Taylor expansion at the previous iterate $\mathbf{W}_k^{(t-1)}$ provides a global affine upper bound. Replacing $g_k(\mathbf{W}_k)$ with this bound gives a concave lower-bound surrogate of the objective (18). This step follows the MM interpretation of the SCA framework, where a concave lower-bound surrogate of the objective is iteratively optimized around the previous iterate. The following explains this process.

Lemma 2. For any concave g_k and any reference point $\mathbf{W}_k^{(t-1)}$, the first order Taylor expansion

$$\begin{aligned} \hat{g}_k(\mathbf{W}_k | \mathbf{W}_k^{(t-1)}) &\triangleq g_k(\mathbf{W}_k^{(t-1)}) \\ &\quad + \langle \nabla_{\mathbf{W}_k^{(t-1)}} g_k(\mathbf{W}_k^{(t-1)}), \mathbf{W}_k - \mathbf{W}_k^{(t-1)} \rangle \end{aligned} \quad (20)$$

is a global upper bound of g_k , i.e.,

$$g_k(\mathbf{W}_k) \leq \hat{g}_k(\mathbf{W}_k | \mathbf{W}_k^{(t-1)}), \quad \forall \mathbf{W}_k \succeq \mathbf{0}. \quad (21)$$

Proof: The inequality follows from the global upper bound property of the supporting hyperplane of a concave function. ■

Substituting the affine upper bound of $g_k(\mathbf{W}_k)$ resulting from Lemma 2 into the objective function in (18) yields the following concave surrogate objective

$$\tilde{\Psi}(\mathcal{W}) \triangleq \sum_{k \in \mathcal{K}} \left[f_k(\mathbf{W}_k) - \hat{g}_k(\mathbf{W}_k | \mathbf{W}_k^{(t-1)}) \right]. \quad (22)$$

Proposition 1. Let $\mathcal{W}^{(t)}$ be the maximizer of (22) at iteration t . Then the sequence produced by the MM updates satisfies

$$\tilde{\Psi}(\mathcal{W}^{(t)}) \geq \tilde{\Psi}(\mathcal{W}^{(t-1)}), \quad \forall t \geq 1, \quad (23)$$

therefore, the objective in (22) is non-decreasing.

Proof: Given (21), the surrogate $\tilde{\Psi}(\mathcal{W} | \mathcal{W}^{(t-1)}) = \sum_{k \in \mathcal{K}} [f_k(\mathbf{W}_k) - \hat{g}_k(\mathbf{W}_k | \mathbf{W}_k^{(t-1)})]$ is a concave lower bound (minorizer) of $\Psi(\mathcal{W})$ that is tight at $\mathcal{W}^{(t-1)}$. More specifically,

$$\tilde{\Psi}(\mathcal{W} | \mathcal{W}^{(t-1)}) \leq \Psi(\mathcal{W}), \quad (24)$$

$$\tilde{\Psi}(\mathcal{W}^{(t-1)} | \mathcal{W}^{(t-1)}) = \Psi(\mathcal{W}^{(t-1)}). \quad (25)$$

By definition of $\mathcal{W}^{(t)}$ as the maximizer of the surrogate at

iteration t , we have

$$\tilde{\Psi}(\mathcal{W}^{(t)} | \mathcal{W}^{(t-1)}) \geq \tilde{\Psi}(\mathcal{W}^{(t-1)} | \mathcal{W}^{(t-1)}). \quad (26)$$

which proves (23). $\blacksquare$

Proposition 1 guarantees that every MM step improves (or leaves unchanged) the surrogate objective in (18). This monotonicity property will be used in the convergence discussion in Proposition 2.

Using E_k 's independence from $\mathbf{W}_k$, the gradient of the function $g_k(\mathbf{W}_k)$ evaluated at iteration $\mathbf{W}_k^{(t-1)}$ yields

$$\nabla_{\mathbf{W}_k^{(t-1)}} g_k(\mathbf{W}_k^{(t-1)}) = \frac{\mu_k^{(t-1)}}{\sqrt{L_k^{(t-1)} + E_k^{(t-1)}}} \mathbf{Z}_e. \quad (27)$$

where $L_k^{(t-1)} = \text{Tr}(\mathbf{Z}_e \mathbf{W}_k^{(t-1)})$, and $E_k^{(t-1)} = \sum_{j \neq k} \text{Tr}(\mathbf{Z}_e \mathbf{W}_j^{(t-1)}) + \sigma_e^2$. Overall, according to Lemma 2, for any concave term $g_k(\mathbf{W}_k)$, its first-order Taylor expansion at $\mathbf{W}_k^{(t-1)}$ provides a global affine upper bound [37]. Substituting this bound in (18) results in the concave surrogate in (22), where the sum of a concave function and an affine function remains concave. Moreover, maximizing this concave surrogate under convex constraints is a convex optimization. Therefore, we can express the BS precoder optimization problem as follows

$$\begin{aligned} & \underset{\{\mathbf{W}_k \succeq \mathbf{0}\}_{k \in \mathcal{K}}}{\text{maximize}} && \tilde{\Psi}(\mathcal{W}) \\ & \text{subject to} && \text{C1} : \sum_{k \in \mathcal{K}} \text{Tr}(\mathbf{W}_k) \leq P \\ & && \text{C3} : \text{rank}(\mathbf{W}_k) = 1. \end{aligned} \quad (28)$$

Since C3 is nonconvex, it is temporarily relaxed using the SDR technique. After each SDP subproblem is solved, we update the iterate, recompute the auxiliary scalars ρ_k and μ_k via their closed form expression presented in Lemma 1, rebuild the linearization in (22), and repeat until the objective increment falls below a preset tolerance. Once the sequence converges, feasible rank-1 precoder vector $\mathbf{w}_k$ is recovered from the optimal precoder matrix by Gaussian randomization followed by power normalization in order to match the requirement of C1 [38]. The summary of the procedure for solving problem (28) is presented in **Algorithm 1**.

Proposition 2. *For the SDR-relaxed version of problem (28) the SCA updates generate the precoder sequence $\mathcal{W}$ for which the surrogate objective values $\tilde{\Psi}(\mathcal{W})$ are monotonically non-decreasing and therefore convergent. Furthermore, every limit point of $\mathcal{W}^{(t)}$ is a stationary point of the SDR-relaxed surrogate objective.*

Proof: According to Proposition 1, each MM/SCA step maximizes a concave lower-bound surrogate that is tight at $\mathcal{W}^{(t-1)}$. The SDR-relaxed feasible set is closed and bounded, and the surrogate $\tilde{\Psi}(\mathcal{W})$ is continuous, so $\tilde{\Psi}(\mathcal{W}^{(t)})$ is bounded above. By monotonicity, this sequence is non-decreasing, and therefore it converges. Furthermore, since the feasible set is compact, the sequence $\mathcal{W}^{(t)}$ admits at least one limit point. Therefore, according to the standard MM convergence results in [39], every limit point of $\mathcal{W}^{(t)}$ satisfies the first-order optimality conditions of the SDR-relaxed version of problem (28) and is therefore a stationary

point. $\blacksquare$

Remark. Lemma 1 shows the auxiliaries ρ_k, μ_k are deterministic functions of the current S_k, I_k, L_k, E_k scalars and hence of the current precoder matrices $\mathbf{W}_k, \forall k \in \mathcal{K}$. In Algorithm 1, we therefore compute $\rho_k^{(t)}$ and $\mu_k^{(t)}$ from $\mathbf{W}_k^{(t-1)}$ via Lemma 1 before solving problem (28). Therefore, no optimization over ρ_k, μ_k is performed inside problem (28).

Optimizing the RIS phase shifter: With the precoder matrix $\mathbf{W}_k$ held fixed, we focus on optimizing the phase shifter vector $\mathbf{v}_R$ of the functional RIS elements while the faulty elements remain as set. A unity slack variable is appended and $\mathbf{v}' \triangleq [\mathbf{v}_R^T \ 1]^T$ is defined. The lifted matrix variable $\mathbf{V} \triangleq \mathbf{v}' \mathbf{v}'^H \in \mathbb{C}^{(\bar{N}+1) \times (\bar{N}+1)}$ is Hermitian, psd, and rank-1, where $\bar{N}$ denotes the number of functioning RIS elements. This lifting step allows the quadratic dependence on the RIS phase shifter vector to be expressed in a linear matrix form, which enables the use of SDP techniques. By expressing each quadratic term in (11) with $\mathbf{V}$, the phase shifter subproblem can be formulated as a SDP solvable with standard convex optimization techniques. Furthermore, we introduce $\mathbf{F}_k$ and $\mathbf{G}_k$ as follows

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{H}_{R,k} \\ \mathbf{h}_{Bd,k}^H \end{bmatrix} \mathbf{w}_k \mathbf{w}_k^H \begin{bmatrix} \mathbf{H}_{R,k}^H & \mathbf{h}_{Bd,k} \end{bmatrix} \in \mathbb{C}^{(\bar{N}+1) \times (\bar{N}+1)} \quad (29)$$

$$\mathbf{G}_k = \begin{bmatrix} \mathbf{H}_{R,e} \\ \mathbf{h}_{Bd,e}^H \end{bmatrix} \mathbf{w}_k \mathbf{w}_k^H \begin{bmatrix} \mathbf{H}_{R,e}^H & \mathbf{h}_{Bd,e} \end{bmatrix} \in \mathbb{C}^{(\bar{N}+1) \times (\bar{N}+1)} \quad (30)$$

Similar to (14), the SINR expressions in terms of $\mathbf{V}$, $\mathbf{F}_k$, and $\mathbf{G}_k$ while keeping $\mathbf{W}_k$ fixed are expressed as follows

$$\begin{aligned} \gamma_k(\mathbf{V}) &= \frac{\text{Tr}(\mathbf{F}_k \mathbf{V})}{\sum_{j \neq k} \text{Tr}(\mathbf{F}_j \mathbf{V}) + \sigma_k^2}, \\ \gamma_e(\mathbf{V}) &= \frac{\text{Tr}(\mathbf{G}_k \mathbf{V})}{\sum_{j \neq k} \text{Tr}(\mathbf{G}_j \mathbf{V}) + \sigma_e^2}, \end{aligned} \quad (31)$$

Therefore, similar to the precoder optimization, we can write the SSR expression for the phase shifter matrix optimization problem, as follows

$$\Upsilon(\mathbf{V}) = \sum_{k \in \mathcal{K}} \frac{1}{\ln 2} \left[\ln \left(1 + \frac{\text{Tr}(\mathbf{F}_k \mathbf{V})}{\sum_{j \neq k} \text{Tr}(\mathbf{F}_j \mathbf{V}) + \sigma_k^2} \right) - \ln \left(1 + \frac{\text{Tr}(\mathbf{G}_k \mathbf{V})}{\sum_{j \neq k} \text{Tr}(\mathbf{G}_j \mathbf{V}) + \sigma_e^2} \right) \right] \quad (32)$$

Similar to (15), we define the following

$$\begin{aligned} A_k &= \text{Tr}(\mathbf{F}_k \mathbf{V}), & B_{j,k} &= \sum_{j \neq k} \text{Tr}(\mathbf{F}_j \mathbf{V}) + \sigma_k^2, \\ C_k &= \text{Tr}(\mathbf{G}_k \mathbf{V}), & D_{j,k} &= \sum_{j \neq k} \text{Tr}(\mathbf{G}_j \mathbf{V}) + \sigma_e^2. \end{aligned} \quad (33)$$

Now by substituting (33) in (32), we obtain

$$\Upsilon(\mathbf{V}) = \sum_{k \in \mathcal{K}} \frac{1}{\ln 2} \left[\ln \left(\frac{A_k + B_{j,k}}{B_{j,k}} \right) - \ln \left(\frac{C_k + D_{j,k}}{D_{j,k}} \right) \right] \quad (34)$$

where the constant factor $\frac{1}{\ln 2}$ uniformly scales the objective and is therefore omitted in this subproblem without affecting the optimizer. Lastly, applying Lemma 1 and Corollary 1 to

(34), with auxiliary scalars $\lambda_k > 0$ and $\eta_k > 0$ yields

$$\Upsilon(\mathbf{V}) = \sum_{k \in \mathcal{K}} \left[2\lambda_k \sqrt{A_k + B_{j,k}} - \lambda_k^2 B_{j,k} - (2\eta_k \sqrt{C_k + D_{j,k}} - \eta_k^2 D_{j,k}) \right], \quad (35)$$

where, for any fixed phase shifter matrix $\mathbf{V}$, the auxiliaries have the closed-form expression defined as

$$\lambda_k = \sqrt{\frac{A_k + B_{j,k}}{B_{j,k}}}, \quad \eta_k = \sqrt{\frac{C_k + D_{j,k}}{D_{j,k}}}. \quad (36)$$

Let us define $\Upsilon(\mathbf{V}) = \sum_{k \in \mathcal{K}} [u_k(\mathbf{V}) - q_k(\mathbf{V})]$ where both $u_k(\mathbf{V})$ and $q_k(\mathbf{V})$ are concave functions of $\mathbf{V}$. Since q_k is concave, its first-order Taylor expansion at $\mathbf{V}^{(t-1)}$ provides an affine upper bound (cf. Lemma 2). Linearizing q_k at $\mathbf{V}^{(t-1)}$ via the SCA procedure yields the following concave minorizer [37]

$$\tilde{\Upsilon}(\mathbf{V}) = \sum_{k \in \mathcal{K}} \left[u_k(\mathbf{V}) - q_k(\mathbf{V}^{(t-1)}) - \langle \nabla_{\mathbf{V}^{(t-1)}} q_k(\mathbf{V}^{(t-1)}), \mathbf{V} - \mathbf{V}^{(t-1)} \rangle \right]. \quad (37)$$

The gradient of $q_k(\mathbf{V})$ with respect to $\mathbf{V}$ evaluated at $\mathbf{V}^{(t-1)}$ is calculated as

$$\nabla_{\mathbf{V}^{(t-1)}} q_k(\mathbf{V}^{(t-1)}) = \frac{\eta_k^{(t-1)}}{\sqrt{\xi_k^{(t-1)}}} (\mathbf{G}_k + \mathbf{G}_{\Sigma,k}) - (\eta_k^{(t-1)})^2 \mathbf{G}_{\Sigma,k}, \quad (38)$$

where $C_k^{(t-1)} = \text{Tr}(\mathbf{G}_k \mathbf{V}^{(t-1)})$ and $D_{j,k}^{(t-1)} = \sum_{j \neq k} \text{Tr}(\mathbf{G}_j \mathbf{V}^{(t-1)}) + \sigma_e^2$ are used for computing $\eta_k^{(t-1)}$. Moreover, $\xi_k^{(t-1)} = C_k^{(t-1)} + D_{j,k}^{(t-1)}$ and $\mathbf{G}_{\Sigma,k} = \sum_{j \neq k} \mathbf{G}_j$. By substituting (38) in (37) the phase shifter optimization subproblem can be expressed as

$$\begin{aligned} & \underset{\mathbf{V} \succeq \mathbf{0}}{\text{maximize}} \quad \tilde{\Upsilon}(\mathbf{V}) \\ & \text{subject to} \quad \overline{\mathbf{C2}} : \text{diag}(\mathbf{V}) = \mathbf{1}, \\ & \quad \quad \quad \mathbf{C4} : \text{rank}(\mathbf{V}) = 1. \end{aligned} \quad (39)$$

where $\overline{\mathbf{C2}}$ is convex and $\mathbf{C4}$ is temporarily relaxed via SDR and later a rank-1 $\mathbf{v}_r$ is extracted by Gaussian randomization [38].

Proposition 3. *For the SDR-relaxed version of problem (39), and with the auxiliary updates in (36), the MM/SCA iterations generate a sequence $\{\mathbf{V}^{(t)}\}$ for which the surrogate objective values $\tilde{\Upsilon}(\mathbf{V}^{(t)})$ are monotonically non-decreasing and thus convergent. Moreover, every limit point of $\{\mathbf{V}^{(t)}\}$ is a stationary point of the SDR-relaxed surrogate objective.*

Proof: As shown in (37), $\tilde{\Upsilon}(\mathbf{V})$ is a concave lower-bound surrogate (minorizer) of $\Upsilon(\mathbf{V})$ that is tight at $\mathbf{V}^{(t-1)}$. At iteration t , the MM/SCA step maximizes this surrogate over the SDR-relaxed feasible set, which implies

$$\tilde{\Upsilon}(\mathbf{V}^{(t)}) \geq \tilde{\Upsilon}(\mathbf{V}^{(t-1)}).$$

Since the feasible set is closed and bounded, and $\tilde{\Upsilon}$ is continuous, the monotone sequence $\{\tilde{\Upsilon}(\mathbf{V}^{(t)})\}$ is bounded above and therefore convergent. Compactness of the feasible set guarantees the existence of at least one limit point of $\{\mathbf{V}^{(t)}\}$. Under the standard MM assumptions (the surrogate is a tight lower bound that shares the first-order behavior of Υ at

the current iterate), every limit point of $\{\mathbf{V}^{(t)}\}$ is a stationary point of the SDR-relaxed problem [39]. ■

The overall AO algorithm for solving (11) is summarized in **Algorithm 1**. According to Proposition 2 and Proposition 3, both surrogate MM objective functions in (28) and (39) are monotonically non-decreasing. Moreover, since the SSR in (11) is evaluated under a transmit power constraint, it is upper-bounded. Consequently, under the standard MM conditions, both surrogate objective values in (28) and (39) converge, and every limit point of the generated iterates is a stationary point of the corresponding SDR-relaxed problem.

Furthermore, since the precoder subproblem in (28) involves a single quadratic constraint (C1), it falls into the class of single-constraint quadratically constrained quadratic programs (QCQPs), where SDR is known to be exact or near-exact [40]. The RIS phase-shifter subproblem in (39) is a complex constant-modulus QCQP, for which SDR has a worst-case approximation ratio of $\pi/4$ [38]. Together, this gives a constant-factor approximation guarantee at the subproblem level.

It is important to emphasize that the above convergence result applies to the surrogate objective defined over the SDR-relaxed feasible set. The subsequent Gaussian randomization step is employed only to extract feasible rank-1 solutions from the SDR matrices and is heuristic in nature. Therefore, the convergence guarantee pertains to the SDR-relaxed optimization procedure and does not extend to the post-randomization original SSR objective. The adopted rank-1 recovery via Gaussian randomization is a widely used approach in SDR-based designs for wireless systems and has been extensively studied in the literature [38]. The behavior of the original SSR objective after rank-1 recovery can be empirically observed through the SSR-versus-iteration curve presented in Fig. 3.

It is worth clarifying the roles of the optimization techniques employed in FIERSE. More specifically, the nonconvex secrecy rate expressions in (16) and (32) are handled using an MM-based SCA procedure. Specifically, MM is used to construct convex surrogate functions that locally approximate the non-convex objective while remaining tight at the current iterate. Solving these surrogate problems sequentially corresponds to the SCA procedure. However, even after applying the MM/SCA transformations, the resulting precoder and RIS phase shifter design subproblems still contain rank-1 constraints due to the lifted variables (e.g., $\mathbf{Q}_k$ and $\mathbf{V}$), which lead to nonconvex QCQPs. To address these rank constraints, SDR is employed to relax the rank-1 conditions and convert the problems into convex semidefinite programs. After solving the SDR-relaxed problems, Gaussian randomization is used to recover feasible rank-1 solutions.

B. Max-min SLNR

In this part, we propose a framework that targets fairness by maximizing the minimum SLNR across UEs. The SLNR quantifies how well each UE's desired signal power is maintained relative to the information leakage to the eavesdropper. Unlike SSR maximization, which can allocate most power to the strongest UE and starve weak ones, max-min SLNR approach equalizes the UEs' link quality by improving the weakest SLNR while suppressing signal leakage. Although max-min SLNR does not directly optimize secrecy rate, increasing each UE SLNR and penalizing leakage to the eavesdropper

enhances the worst UE's secrecy performance. Under *perfect* CSI, the resulting max-min SLNR problem is formulated as two alternating SDPs: one for the BS precoder and another for the RIS phase shift matrix. Both subproblems can be efficiently solved as classical max-min optimization problems [41]. For the *statistical* CSI case, we tighten the minimum average SLNR using Jensen's inequality and reformulate the maximization as a feasibility problem that can be solved iteratively. Both perfect-CSI and statistical-CSI variants optimize faulty RIS-assisted networks by suppressing leakage and maximizing worst-UE SLNR. The perfect-CSI variant provides an achievable performance upperbound, while the statistical-CSI variant offers practical robustness when only the knowledge of the location of faulty elements is available. Furthermore, similar to the FIERSE scheme, the perfect CSI assumption is adopted primarily to focus on the paper's core objective, namely the impact of faulty RIS elements on secrecy performance and the design of robust fault-resilient frameworks. In the following, we outline perfect-CSI and statistical-CSI solutions in detail.

Algorithm 1 FIERSE algorithm for solving problem (11)

- 1: **Input:** Initialize $\mathbf{V}^{(0)}$, $\rho_k^{(0)}$, $\mu_k^{(0)}$, $\lambda_k^{(0)}$, $\eta_k^{(0)}$, $\mathbf{W}_k^{(0)}$. Set iteration counter $t = 0$, tolerance ϵ
 - 2: **repeat**
 - 3: **Step 1: Optimization of $\mathbf{w}_k$ based on a fixed $\mathbf{v}_R$:**
 - 1.1 Given $\mathbf{V}^{(t)}$, $\rho_k^{(t)}$, and $\mu_k^{(t)}$ solve (28).
 - 1.2 Update $\mathbf{W}_k^{(t+1)}$, $\rho_k^{(t+1)}$, and $\mu_k^{(t+1)}$ using the solution of step 1.1
 - 4: **Step 2: Optimization of $\mathbf{V}$ based on fixed $\mathbf{W}_k$:**
 - 2.1 Given $\mathbf{W}_k^{(t)}$, $\lambda_k^{(t)}$, and $\eta_k^{(t)}$ solve (39).
 - 2.2 Update $\mathbf{V}^{(t+1)}$, $\lambda_k^{(t+1)}$, and $\eta_k^{(t+1)}$ using the solution of step 2.1
 - 5: Using Gaussian randomization, draw $L = 1000$ i.i.d. samples, project each sample onto the feasibility set, evaluate the objective, and keep the best one.
 - 6: $t \leftarrow t + 1$
 - 7: **Until** $\frac{\sum_{k \in \mathcal{K}} (R_{s,k}^{(t)} - R_{s,k}^{(t-1)})}{\sum_{k \in \mathcal{K}} R_{s,k}^{(t-1)}} \leq \epsilon$
 - 8: **Output:** $\{\mathbf{w}_k^*, \mathbf{v}_R^*\}$
-

1) *Perfect CSI:* Given the SLNR definition in (9), we formulate the max-min SLNR problem as follows

$$\begin{aligned} & \underset{\mathbf{w}_k, \mathbf{v}_R}{\text{maximize}} && \min_k \text{SLNR}_k \\ & \text{subject to} && \text{C1, C2.} \end{aligned} \quad (40)$$

Due to the interdependence of two variables, $\mathbf{w}_k$ and $\mathbf{v}_R$, in the objective function of (40), and the presence of the unit-modulus constraint in C2, problem (40) is inherently nonconvex. To address this, we will optimize the problem alternatively, focusing on each variable, $\mathbf{w}_k$ and $\mathbf{v}_R$, separately. Each resulting subproblem can be reduced to a well-studied form [42]. To address the nonconvexity, we employ an SDR approach and solve the resulting feasibility problem using bisection method. We subsequently solve the resulting problem using the CVX solver [43] [44]. In the following, we detail the solution for the precoder and phase shifter separately.

Optimizing the precoder at the BS: Here we optimize the BS precoder vector $\mathbf{w}_k$ of each legitimate UE k , while keeping the phase shifter vector at the RIS fixed. By defin-

ing $\mathbf{L}_k \triangleq \mathbf{w}_k \mathbf{w}_k^H$ as a Hermitian matrix, we can rewrite (40) as an SDP problem with respect to $\mathbf{L}_k$ and convert it into a feasibility problem by introducing the slack variable $\beta_1^p = \min \text{SLNR}_k$. Moreover, we have $\|\mathbf{w}_k\|^2 = \text{Tr}(\mathbf{L}_k)$, therefore we can translate (40) to the following problem

$$\begin{aligned} & \underset{\mathbf{L}_k \succeq \mathbf{0}, \beta_1^p \geq 0}{\text{maximize}} && \beta_1^p \\ & \text{subject to} && \widetilde{\text{C1}} : \sum_{k \in \mathcal{K}} \text{Tr}(\mathbf{L}_k) \leq P \\ & && \text{C5} : \text{Tr}(\mathbf{L}_k \mathbf{H}_k^p) \geq \beta_1^p [\text{Tr}(\mathbf{L}_k \mathbf{H}_e^p) + \sigma_e^2], \quad \forall k \\ & && \text{C6} : \text{rank}(\mathbf{L}_k) = 1, \quad \forall k, \end{aligned} \quad (41)$$

where for each $i \in \{e, k\}$

$$\mathbf{H}_i^p = (\mathbf{v}_R^H \bar{\mathbf{H}}_{R,i} + \mathbf{h}_{Bd,i}^H)^H (\mathbf{v}_R^H \bar{\mathbf{H}}_{R,i} + \mathbf{h}_{Bd,i}^H) \in \mathbb{C}^{M \times M}. \quad (42)$$

Since (41) remains nonconvex because of C6, we apply SDR, relax C6 and solve the resulting convex feasibility problem via bisection on β_1^p . Subsequently, the precoder $\mathbf{w}_k$ is extracted from $\mathbf{L}_k$ using Gaussian randomization.

Optimizing the phase shifter at the RIS: For a fixed precoder $\mathbf{w}_k$, (40) reduces to finding the optimal RIS phase shift vector $\mathbf{v}_R$. By lifting the vector into a rank-1 matrix $\tilde{\mathbf{v}} \triangleq [\mathbf{v}_R^T \ 1]^T$, and defining $\tilde{\mathbf{V}} \triangleq \tilde{\mathbf{v}} \tilde{\mathbf{v}}^H \in \mathbb{C}^{(\bar{N}+1) \times (\bar{N}+1)}$ we have

$$\begin{aligned} & \underset{\tilde{\mathbf{V}} \succeq \mathbf{0}, \beta_2^p \geq 0}{\text{maximize}} && \beta_2^p \\ & \text{subject to} && \widetilde{\text{C2}} : \text{diag}(\tilde{\mathbf{V}}) = \mathbf{1}, \\ & && \text{C7} : \text{Tr}(\tilde{\mathbf{H}}_k^p \tilde{\mathbf{V}}) \geq \beta_2^p [\text{Tr}(\tilde{\mathbf{H}}_e^p \tilde{\mathbf{V}}) + \sigma_e^2], \quad \forall k, \\ & && \text{C8} : \text{rank}(\tilde{\mathbf{V}}) = 1, \end{aligned} \quad (43)$$

where β_2^p is a slack variable that represents the minimum SLNR across all legitimate UEs in (43). Furthermore, for each $i \in \{e, k\}$ we define $\tilde{\mathbf{H}}_i^p$ as follows

$$\tilde{\mathbf{H}}_i^p \triangleq \begin{bmatrix} \bar{\mathbf{H}}_{R,i} \\ \mathbf{h}_{Bd,i}^H \end{bmatrix} \mathbf{w}_k \mathbf{w}_k^H \begin{bmatrix} \bar{\mathbf{H}}_{R,i}^H & \mathbf{h}_{Bd,i} \end{bmatrix} \in \mathbb{C}^{(\bar{N}+1) \times (\bar{N}+1)}. \quad (44a)$$

Constraints $\widetilde{\text{C2}}$, C7, and C8 enforce unit-modulus elements, the SLNR-driven linear matrix inequalities, and the rank-1 requirement on $\tilde{\mathbf{V}}$, respectively. Problem (43) can be solved using CVX. Specifically, we first drop the nonconvex constraint C8, solve the resulting SDP using bisection on β_2^p , and then restore the rank constraint through Gaussian randomization. The complete AO algorithm that solves (40) is presented in **Algorithm 2**.

2) *Statistical CSI:* In practice, obtaining perfect CSI for all RIS links is often infeasible due to the high channel estimation overhead and hardware limitations. To address this, we consider a more practical setting where only statistical CSI is available. This variant relaxes the perfect CSI assumption and relies solely on knowledge of the faulty elements' locations, without requiring their exact phase shifting values. While this design assumes knowledge of fault locations and second-order channel statistics, these parameters vary on a much slower timescale than instantaneous channel fading. In practical deployments, fault locations are acquired via established low-overhead diagnostics such as compressed sensing [24] or pilot-based detection [25], which remain valid over

long operating intervals. Similarly, second-order statistics such as path loss and Rician K-factors are readily derived from long-term measurements [45]. Consequently, this approach eliminates the high overhead of tracking the instantaneous random behavior of faulty elements within every coherence block, offering a robust solution when precise channel estimation is infeasible. The following outlines the details of this statistical-CSI approach.

Algorithm 2 Maxmin-perfect algorithm for solving (40)

```

1: Set iteration counter  $t = 0$ , tolerance  $\delta$ , convergence
   function  $\mathcal{J}^{(0)} = -\infty$ , and the slack variables  $\beta_1^{\text{p,max}}$ ,
    $\beta_1^{\text{p,min}}$ ,  $\beta_2^{\text{p,max}}$ , and  $\beta_2^{\text{p,min}}$  to feasible bounds. Initialize
   the phase shifter  $\mathbf{v}_R^{(t)}$ ,
2: repeat ▷ Outer AO loop
   1. Precoder update: solve (41)
   3: while  $(\beta_1^{\text{p,max}} - \beta_1^{\text{p,min}})/\beta_1^{\text{p,max}} > \delta$  do
   4:    $\beta_1^{\text{p}} \leftarrow (\beta_1^{\text{p,max}} + \beta_1^{\text{p,min}})/2$ 
   5:   if (41) is feasible with  $\mathbf{v}_R^{(t)}$  and  $\beta_1^{\text{p}}$  then
   6:      $\beta_1^{\text{p,min}} \leftarrow \beta_1^{\text{p}}$ 
   7:   else
   8:      $\beta_1^{\text{p,max}} \leftarrow \beta_1^{\text{p}}$ 
   9:   end if
10: end while
11:  $\mathbf{w}_k^{(t)} \leftarrow$  Gaussian-randomize SDR solution
   2. RIS phase update: solve (43)
12: while  $(\beta_2^{\text{p,max}} - \beta_2^{\text{p,min}})/\beta_2^{\text{p,max}} > \delta$  do
13:    $\beta_2^{\text{p}} \leftarrow (\beta_2^{\text{p,max}} + \beta_2^{\text{p,min}})/2$ 
14:   if (43) is feasible with  $\mathbf{w}_k^{(t)}$  and  $\beta_2^{\text{p}}$  then
15:      $\beta_2^{\text{p,min}} \leftarrow \beta_2^{\text{p}}$ 
16:   else
17:      $\beta_2^{\text{p,max}} \leftarrow \beta_2^{\text{p}}$ 
18:   end if
19: end while
20:  $\mathbf{v}_R^{(t+1)} \leftarrow$  Gaussian-randomize SDR solution
21:  $\mathcal{J}^{(t+1)} \leftarrow \min_k \text{SLNR}(\mathbf{w}_k^{(t)}, \mathbf{v}_R^{(t+1)})$ 
22:  $t \leftarrow t + 1$ 
23: until  $|\mathcal{J}^{(t+1)} - \mathcal{J}^{(t)}|/|\mathcal{J}^{(t+1)}| < \delta$ 
24: return  $\mathbf{w}_k^* = \mathbf{w}_k^{(t)}$ ,  $\mathbf{v}_R^* = \mathbf{v}_R^{(t+1)}$ 

```

Average minimum SLNR design: To obtain a tractable objective in the presence of faulty RIS elements, we first average the instantaneous SLNR in (9) with respect to the faulty-element vector $\mathbf{v}_B$ and gain a lower bound for the SLNR expression. Later, we adopt this bound as the performance metric and formulate a max-min design that maximizes the worst-UE average SLNR subject to the transmit power and RIS constraints.

Lemma 3 (Average SLNR lower bound). *Assuming the channel blocks $(\bar{\mathbf{H}}_{R,k}, \bar{\mathbf{H}}_{B,k})$ and $(\bar{\mathbf{H}}_{R,e}, \bar{\mathbf{H}}_{B,e})$ are statistically independent (e.g., independent Rician blocks), and by defining*

$$\begin{aligned} \mathbf{h}_k^{\text{eff}} &\triangleq (\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} + \mathbf{v}_B^H \bar{\mathbf{H}}_{B,k}) \mathbf{w}_k, \\ \mathbf{h}_e^{\text{leak}} &\triangleq (\mathbf{v}_R^H \bar{\mathbf{H}}_{R,e} + \mathbf{v}_B^H \bar{\mathbf{H}}_{B,e}) \mathbf{w}_k, \end{aligned} \quad (45)$$

using $\mathbf{h}_{Bd,i}^H = \mathbf{v}_B^H \bar{\mathbf{H}}_{B,i}$ for $i \in \{k, e\}$, the expected SLNR

satisfies

$$\mathbb{E}[\text{SLNR}_k] \geq \frac{\mathbb{E}[|\mathbf{h}_k^{\text{eff}}|^2]}{\mathbb{E}[|\mathbf{h}_e^{\text{leak}}|^2] + \sigma_e^2}. \quad (46)$$

Proof: After averaging over $\mathbf{v}_B$, the numerator and denominator of SLNR_k depend on the independent channel blocks $(\bar{\mathbf{H}}_{R,k}, \bar{\mathbf{H}}_{B,k})$ and $(\bar{\mathbf{H}}_{R,e}, \bar{\mathbf{H}}_{B,e})$, respectively. Let

$$X \triangleq |\mathbf{h}_k^{\text{eff}}|^2 \geq 0, \quad Z \triangleq |\mathbf{h}_e^{\text{leak}}|^2 + \sigma_e^2 > 0. \quad (47)$$

Then X and Z are independent, and

$$\mathbb{E}\left[\frac{X}{Z}\right] = \mathbb{E}[X] \mathbb{E}\left[\frac{1}{Z}\right] \geq \frac{\mathbb{E}[X]}{\mathbb{E}[Z]}. \quad (48)$$

where the inequality uses Jensen's inequality for the convex function $f(z) = 1/z$ (hence $\mathbb{E}[1/Z] \geq 1/\mathbb{E}[Z]$). Substituting X and Z yields the lower bound in (46). ■

We subsequently use (46) as the performance metric to be maximized [17].

As outlined in Sec. II, we model the effect caused by the faulty elements as a phase shift with $v_{B,b} = \delta_b e^{j\phi_b}$, $b = 1, \dots, B$, with the amplitude $\delta_b \sim \mathcal{U}(0, 1)$ and the phase shifter $\phi_b \sim \mathcal{U}[0, 2\pi)$ [17]. Thus, by defining $\mathbf{x}_{B,k} = \bar{\mathbf{H}}_{B,k} \mathbf{w}_k \in \mathbb{C}^{B \times 1}$ we have

$$\begin{aligned} \mathbb{E}[|\mathbf{v}_B^H \mathbf{x}_{B,k}|^2] &= \mathbb{E}[\delta_b^2] \|\bar{\mathbf{H}}_{B,k} \mathbf{w}_k\|^2 \\ &= \left(\frac{1}{1-0} \int_0^1 \delta_b^2 d\delta_b\right) \|\bar{\mathbf{H}}_{B,k} \mathbf{w}_k\|^2 \\ &= \frac{1}{3} \|\bar{\mathbf{H}}_{B,k} \mathbf{w}_k\|^2 \end{aligned} \quad (49)$$

Note that if the amplitude δ_b in (49) follows a different distribution, such as a Rayleigh distribution, or if it is a fixed attenuated constant, the resulting value of $\mathbb{E}[\delta_b^2]$ changes accordingly. In particular, for $\delta_b \sim \text{Rayleigh}(\sigma)$, where σ denotes the scale parameter controlling the average amplitude of the faulty element, we obtain $\mathbb{E}[\delta_b^2] = 2\sigma^2$. Alternatively, if the amplitude is modeled as a constant attenuation $\delta_b = \delta_0$, where $\delta_0 \in (0, 1)$ represents a constant gain reduction of the faulty element, then $\mathbb{E}[\delta_b^2] = \delta_0^2$. Therefore, the derivation in (49) can be generalized to other amplitude distributions by replacing the factor $1/3$ with the corresponding expectation $\mathbb{E}[\delta_b^2]$. Therefore, by using (49) we can simplify $\mathbb{E}[|\mathbf{h}_k^{\text{eff}}|^2]$ as follows

$$\begin{aligned} \mathbb{E}[|\mathbf{h}_k^{\text{eff}}|^2] &= |\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} \mathbf{w}_k|^2 + \mathbb{E}[|\mathbf{v}_B^H \mathbf{x}_{B,k}|^2] \\ &\quad + 2 \Re\left\{(\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} \mathbf{w}_k) \mathbf{x}_{B,k}^H \mathbb{E}[\mathbf{v}_B]\right\} \\ &= |\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} \mathbf{w}_k|^2 + \frac{1}{3} \|\bar{\mathbf{H}}_{B,k} \mathbf{w}_k\|^2, \end{aligned} \quad (50)$$

Since $\mathbb{E}[\mathbf{v}_B] = \mathbf{0}$ (the phase shifter of each faulty element is uniformly distributed), the cross-term in (50) vanishes. Similar to (50), we can derive the following for eavesdropper

$$\begin{aligned} \mathbb{E}_{\mathbf{v}_B}[|\mathbf{h}_e^{\text{leak}}|^2] + \sigma_e^2 &= \\ |\mathbf{v}_R^H \bar{\mathbf{H}}_{R,e} \mathbf{w}_k|^2 + \frac{1}{3} \|\bar{\mathbf{H}}_{B,e} \mathbf{w}_k\|^2 + \sigma_e^2. \end{aligned} \quad (51)$$

By substituting (50) into the numerator and (51) into the denominator of the lower-bound expression in (46), the expected SLNR of the legitimate UE k admits the following deterministic bound

$$\mathbb{E}[\text{SLNR}] \geq \frac{|\mathbf{v}_R^H \bar{\mathbf{H}}_{R,k} \mathbf{w}_k|^2 + \frac{1}{3} \|\bar{\mathbf{H}}_{B,k} \mathbf{w}_k\|^2}{|\mathbf{v}_R^H \bar{\mathbf{H}}_{R,e} \mathbf{w}_k|^2 + \frac{1}{3} \|\bar{\mathbf{H}}_{B,e} \mathbf{w}_k\|^2 + \sigma_e^2}. \quad (52)$$

To streamline the formulation, we use the right-hand side of (52) as the surrogate objective in the max-min SLNR problem and denote it by β^s . Thus, the max-min problem in (40) is reformulated as follows

$$\begin{aligned} & \underset{\mathbf{w}_k, \mathbf{v}_R}{\text{maximize}} \quad \beta^s \\ & \text{subject to} \quad \text{C9: } |\mathbf{v}_R^H \tilde{\mathbf{H}}_{R,k} \mathbf{w}_k|^2 + \frac{1}{3} \|\tilde{\mathbf{H}}_{B,k} \mathbf{w}_k\|^2 \geq \\ & \quad \beta^s \left[|\mathbf{v}_R^H \tilde{\mathbf{H}}_{R,e} \mathbf{w}_k|^2 + \frac{1}{3} \|\tilde{\mathbf{H}}_{B,e} \mathbf{w}_k\|^2 + \sigma_e^2 \right], \forall k \\ & \quad \text{C1, C2.} \end{aligned} \quad (53)$$

where Constraint C9 enforces that, for every legitimate UE, the desired signal power exceeds a scaled version of the eavesdropper's signal plus noise. Since (53) is a nonconvex joint optimization problem due to the multiplicative coupling between $\mathbf{w}_k$ and $\mathbf{v}_R$, we outline below how it can be addressed via AO, i.e., by alternatively optimizing the precoder for a fixed functioning phase shifters and vice versa.

Precoder design: To facilitate the AO design and apply the SDP framework, we first define $\tilde{\mathbf{Q}}_k = \mathbf{w}_k \mathbf{w}_k^H$, $\mathbf{H}_i^{\text{stat}} = (\mathbf{v}_R^H \tilde{\mathbf{H}}_{R,i})^H (\mathbf{v}_R^H \tilde{\mathbf{H}}_{R,i})$, and $\tilde{\mathbf{H}}_i^{\text{stat}} = (\tilde{\mathbf{H}}_{B,i})^H (\tilde{\mathbf{H}}_{B,i}) \in \mathbb{C}^{M \times M}$, $\forall i \in \{e, k\}$. Using these definitions, we reformulate (53) into (54) as follows

$$\begin{aligned} & \underset{\tilde{\mathbf{Q}}_k, \beta_1^s}{\text{maximize}} \quad \beta_1^s \\ & \text{subject to} \quad \text{C10: } \text{Tr}(\tilde{\mathbf{Q}}_k \mathbf{H}_k^{\text{stat}}) + \frac{1}{3} \text{Tr}(\tilde{\mathbf{Q}}_k \tilde{\mathbf{H}}_k^{\text{stat}}) \geq \\ & \quad \beta_1^s \left[\text{Tr}(\tilde{\mathbf{Q}}_k \mathbf{H}_e^{\text{stat}}) + \frac{1}{3} \text{Tr}(\tilde{\mathbf{Q}}_k \tilde{\mathbf{H}}_e^{\text{stat}}) + \sigma_e^2 \right], \forall k \\ & \quad \overline{\text{C1}}: \sum_{k \in \mathcal{K}} \text{Tr}(\tilde{\mathbf{Q}}_k) \leq P \\ & \quad \overline{\text{C3}}: \text{rank}(\tilde{\mathbf{Q}}_k) = 1, \quad \forall k. \end{aligned} \quad (54)$$

where β_1^s is the minimum SLNR value for (54). By adopting the SDR method, we relax the restriction imposed by the rank-1 constraint in $\overline{\text{C3}}$, allowing us to address the problem using the bisection search.

Phase shifter design: Similar to precoder design, by leveraging SDP, we recast (53) to (55) with the purpose of optimizing the phase shifter at the RIS. This reformulation is obtained by introducing $\tilde{\mathbf{V}}_R = \mathbf{v}_R \mathbf{v}_R^H$ and defining $\tilde{\mathbf{H}}_i^s = (\tilde{\mathbf{H}}_{R,i} \mathbf{w}_k)(\tilde{\mathbf{H}}_{R,i} \mathbf{w}_k)^H \in \mathbb{C}^{\tilde{N} \times \tilde{N}}$, $\forall i \in \{e, k\}$, as follows

$$\begin{aligned} & \underset{\tilde{\mathbf{V}}_R \succeq \mathbf{0}, \beta_2^s}{\text{maximize}} \quad \beta_2^s \\ & \text{subject to} \quad \text{C11: } \text{Tr}(\tilde{\mathbf{H}}_k^s \tilde{\mathbf{V}}_R) + \frac{1}{3} \|\tilde{\mathbf{H}}_{B,k} \mathbf{w}_k\|^2 \geq \\ & \quad \beta_2^s \left[\text{Tr}(\tilde{\mathbf{H}}_e^s \tilde{\mathbf{V}}_R) + \frac{1}{3} \|\tilde{\mathbf{H}}_{B,e} \mathbf{w}_k\|^2 + \sigma_e^2 \right], \forall k \\ & \quad \overline{\text{C2}}: \text{diag}(\tilde{\mathbf{V}}_R) = \mathbf{1} \\ & \quad \overline{\text{C8}}: \text{rank}(\tilde{\mathbf{V}}_R) = 1. \end{aligned} \quad (55)$$

where β_2^s corresponds to the minimum SLNR value of (55). The constraint $\overline{\text{C8}}$ adds nonconvexity to problem (55). Therefore, using the SDR approach, we first relax $\overline{\text{C8}}$ and solve (55) using the bisection method. Later, once the solution is derived, we use the Gaussian randomization method to get an

approximate rank-1 solution that satisfies all the constraints in (55). **Algorithm 3** summarizes the whole procedure for solving (53).

Algorithm 3 Maxmin-statistical algorithm for solving (53)

Set iteration counter $t = 0$, tolerance δ , convergence function $\mathcal{J}^{(0)} = -\infty$, and the slack variables $\beta_1^{s,\max}$, $\beta_1^{s,\min}$, $\beta_2^{s,\max}$, and $\beta_2^{s,\min}$ to feasible bounds. Initialize the phase shifter $\mathbf{v}_R^{(t)}$,

repeat $\triangleright$ Outer AO loop

1. Precoder update: solve (54)

while $(\beta_1^{s,\max} - \beta_1^{s,\min})/\beta_1^{s,\max} > \delta$ **do**

$\beta_1^s \leftarrow (\beta_1^{s,\max} + \beta_1^{s,\min})/2$

if (54) is feasible with $\mathbf{v}_R^{(t)}$ and β_1^s **then**

$\beta_1^{s,\min} \leftarrow \beta_1^s$

else

$\beta_1^{s,\max} \leftarrow \beta_1^s$

end if

end while

$\mathbf{w}_k^{(t)} \leftarrow$ Gaussian-randomize SDR solution

2. RIS phase update: solve (55)

while $(\beta_2^{s,\max} - \beta_2^{s,\min})/\beta_2^s > \delta$ **do**

$\beta_2^s \leftarrow (\beta_2^{s,\max} + \beta_2^{s,\min})/2$

if (55) is feasible with $\mathbf{w}_k^{(t)}$ and β_2^s **then**

$\beta_2^{s,\min} \leftarrow \beta_2^s$

else

$\beta_2^{s,\max} \leftarrow \beta_2^s$

end if

end while

$\mathbf{v}_R^{(t+1)} \leftarrow$ Gaussian randomized SDR solution

$\mathcal{J}^{(t+1)} \leftarrow \min_k \text{SLNR}(\mathbf{w}_k^{(t)}, \mathbf{v}_R^{(t+1)})$

$t \leftarrow t + 1$

until $|\mathcal{J}^{(t+1)} - \mathcal{J}^{(t)}|/|\mathcal{J}^{(t+1)}| < \delta$

return $\mathbf{w}_k^* = \mathbf{w}_k^{(t)}$, $\mathbf{v}_R^* = \mathbf{v}_R^{(t+1)}$

C. Complexity analysis

Each outer iteration of **Algorithm 1** solves K SDPs of size $(M \times M)$ for the precoder and one SDP of size $(\tilde{N} + 1) \times (\tilde{N} + 1)$ for the functioning phase shifters of the RIS. Therefore, the whole **Algorithm 1** has a complexity of $\mathcal{O}(I_{\text{AO}} I_{\text{SCA}} ((KM)^{3.5} + \tilde{N}^{3.5}) \log \frac{1}{\epsilon})$, where I_{AO} , and I_{SCA} are the number of iterations for the AO and the SCA schemes, respectively [7]. Similarly, we derive the complexity for **Algorithm 2** and **Algorithm 3** as $\mathcal{O}(I_{\text{AO}}^p ((KM)^{3.5} + \tilde{N}^{3.5}) \log \frac{1}{\delta})$ and $\mathcal{O}(I_{\text{AO}}^s ((KM)^{3.5} + \tilde{N}^{3.5}) \log \frac{1}{\delta})$, respectively, where I_{AO}^p and I_{AO}^s are the iteration numbers for the outer AO loop of **Algorithm 2** and **Algorithm 3**, respectively.

IV. SIMULATION RESULTS

A. Simulation setup

As depicted in Fig. 2, we consider a 3-D Cartesian coordinate system where BS and RIS are centered at $[0, 20, 4]$ and $[0, 0, 4]$, respectively, and the legitimate UEs and the eavesdropper are distributed with a random uniform distribution within the highlighted circular area. We assume that all direct

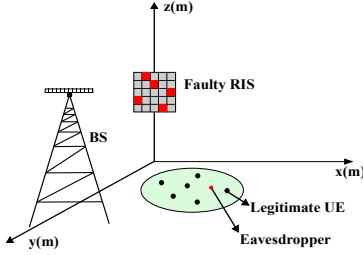


Fig. 2: Simulation setup for the considered secure MU-MISO system with multiple legitimate UEs, an eavesdropper, and a faulty RIS.

BS-UE and BS-eavesdropper links are blocked by obstacles. Therefore, the need for a RIS is essential for providing communication. Furthermore, we consider a worst-case scenario where there is a LoS path between RIS and the eavesdropper. In the simulations, we consider a single eavesdropper modeled as a worst-case adversary with perfect decoding capability and full knowledge of its channel, consistent with the assumption stated in Remark 1.

Our proposed framework is tested and simulated under the assumption of a Rician fading channel, where the effects of both LoS and Non-Line-of-Sight (NLoS) channels are considered. Therefore, we express the channel for the BS-RIS link as follows [17]

$$\mathbf{G} \triangleq \sqrt{\frac{K_{BR}}{1 + K_{BR}}} \mathbf{G}^{\text{LoS}} + \sqrt{\frac{1}{1 + K_{BR}}} \mathbf{G}^{\text{NLoS}}. \quad (56)$$

where K_{BR} , $\mathbf{G}^{\text{LoS}}$, $\mathbf{G}^{\text{NLoS}}$ stand for the Rician factor, the BS-RIS LoS link, and the BS-RIS NLoS link, respectively. Similarly, we have configured the channel for the RIS-UE, and RIS-eavesdropper links with their Rician factors defined as K_{RU} , and K_{RE} , respectively. Moreover, path loss is modeled as a distance-dependent power law $P_L(d_{t,r}) = C_0(d_0/d_{t,r})^\sigma$, where $C_0 = (\lambda_c/(4\pi))^2 = -61$ dB at the reference distance $d_0 = 1$ m, $d_{t,r}$ denotes transmitter-receiver distance, σ is the pathloss exponent, and λ_c is the wavelength [42]. The path loss exponent for the links BS-RIS, RIS-UE, and RIS-eavesdropper are 2, 2.2, and 2 respectively [46], [18].

The performance of our results is tested by Monte Carlo simulation, where in each iteration the location of the legitimate UE and eavesdropper follows a random uniform distribution with the center at (2, 10, 4) and a radius of $r_c = 8$, and the location of faulty RIS elements is drawn at random. Moreover, each faulty element is assigned a fixed phase shift $\mathcal{U}[0, 2\pi)$ and an amplitude $\mathcal{U}(0, 1)$. Furthermore, for each channel, we randomly generate the NLoS scattering paths using $L_{cl} = 5$ clusters, each with $R_{cl} = 20$ subpaths where the cluster volumes are defined as in [46]. The remaining channel parameters are listed in Table I unless stated otherwise.

B. Baseline schemes

In this part, we compare the performance of **Algorithm 1**, **Algorithm 2**, and **Algorithm 3**, referred to as FIERSE, maxmin-perfect, and maxmin-statistical, respectively. We further evaluate these proposed approaches against four baseline schemes [17]:

- 1) *No-fault scheme (upper bound)*: This scheme assumes a fully functional RIS (no faulty elements) and solves the SSR maximization problem accordingly. This serves as an optimistic upper bound against fault-aware designs.
- 2) *Naive scheme*: In this baseline scheme, the RIS phase shifter optimization is performed while ignoring the presence of faulty elements, i.e., the phase shifters are optimized assuming that all RIS elements function normally. However, during performance evaluation, the actual hardware faults remain present, leading to degraded performance due to the mismatch between the assumed and the true RIS responses. For a fair comparison with our two objectives, we introduce two variants: (i) naive, which maximizes SSR (as in FIERSE), and (ii) naive-maxmin, as in the maxmin-perfect scheme.
- 3) *Random scheme*: In random scheme, the BS precoder is optimized, while all RIS elements apply random phase shifts. We implement this scheme with two precoder designs: (i) random, using the FIERSE formulation, (ii) random-maxmin, following the maxmin-perfect design. Due to the absence of coherent RIS beamforming gain, this scheme can be interpreted as approximating a RIS-less baseline analogous to a NLoS scenario.
- 4) *AN scheme*: In the AN scheme, the BS transmits AN together with the information signal to degrade the eavesdropper's reception. This scheme represents a classical PLS technique and serves as a baseline for our comparison.

In the following, we evaluate the performance of our proposed algorithms along with the baselines across different key system parameters, including the number of RIS elements, number of faulty elements, transmit power, and number of legitimate UEs. For clarity, the evaluated schemes can be grouped based on their optimization objective. The schemes *no-fault*, *FIERSE*, *AN*, *random* and *naive* correspond to **SSR-based** designs, whereas *maxmin-perfect*, *maxmin-statistical*, and *random-maxmin* correspond to **SLNR-based** schemes.

TABLE I: Simulation Parameters

Parameter	Value	Parameter	Value
Frequency	28 GHz	K	2
P	24 dBm	ϵ	10^{-3}
σ_n^2	-80 dBm	K_{BR}	10 dB
M	16	K_{RU}	10 dB
N_x	9	K_{RE}	10 dB
N_z	9	δ	10^{-3}
B	50		

C. Convergence behavior of our proposed algorithms

Fig. 3 depicts the SSR against AO iteration number for FIERSE scheme with two different configurations. Both FIERSE curves exhibit strictly monotonic ascent, confirming the monotonic ascent property of the AO framework. In both configurations, the objective increases sharply during the first roughly 10 AO iterations, accounting for most of the improvement, while the improvements become progressively smaller as convergence is approached. As shown, FIERSE with

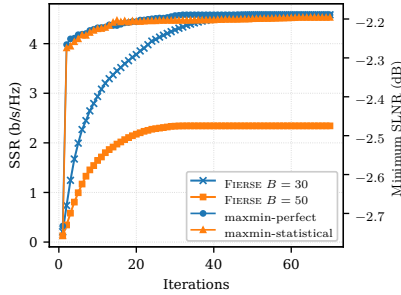


Fig. 3: Convergence of the FIERSE scheme for $B = 50$ and $B = 30$, along with the maxmin-perfect and maxmin-statistical schemes. The curves show the true SSR evaluated at the feasible solution obtained after each AO iteration, where rank-1 solutions are recovered from the SDR subproblems via Gaussian randomization.

$B = 30$ converges to a higher SSR after roughly 50 iterations, whereas FIERSE with $B = 50$ converges to a lower SSR after roughly 30 iterations. The reason for this is that for smaller number of faulty elements, the optimization involves a larger number of functional elements in the phase shifter subproblem, which slows down convergence. However, the presence of more functional elements also allows the system to achieve a higher final SSR value. Fig. 3 also shows the worst-UE SLNR for the max-min schemes under perfect and statistical CSI. Both maxmin-perfect and maxmin-statistical curves exhibit the same pattern where an initial sharp rise takes place within the first roughly 10-15 iterations, followed by small improvements, and convergence in roughly 30 iterations. The maxmin-perfect scheme achieves a slightly higher objective value as it has full channel knowledge, whereas the statistical variant optimizes over statistical CSI, which yields a slightly lower final objective. The monotonic behavior of both curves confirms the convergence of both proposed max-min schemes.

D. Impact of number of RIS elements

As depicted in Fig. 4, all proposed algorithms are compared against each other and the baseline schemes when the number of RIS elements varies. In all schemes, SSR continuously increases with the number of RIS elements, although the slope is the slowest for the random baseline. As expected, no-fault achieves the highest SSR reflecting the upper bound limit. FIERSE begins with a huge gap compared to no-fault, as in the beginning, there are approximately 60 elements, where 50 of them are broken. Therefore, the SSR can be substantially degraded. Nevertheless, FIERSE outperforms the maxmin-perfect, maxmin-statistical, naive, AN, and random. It can be seen that as the number of RIS elements exceeds 160 elements, the performance gap between FIERSE and no-fault reaches its minimum. This happens since the percentage of faulty elements reduces to 30% when $N = 169$. (Note that in this figure the number of faulty elements is fixed at $B = 50$; therefore, enlarging the RIS size reduces the proportion of faulty elements.) Moreover, the graph showing the random baseline confirms the worsened behavior in the case of no RIS configuration, and the AN baseline, although

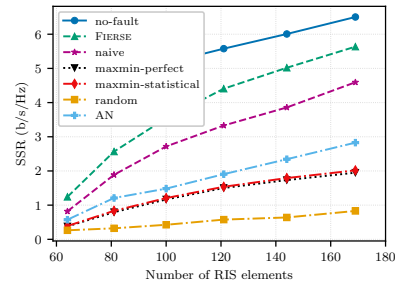


Fig. 4: SSR vs. number of RIS elements.

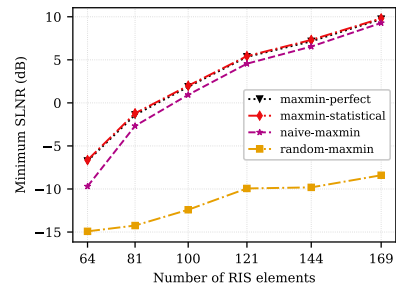


Fig. 5: Minimum SLNR vs. number of RIS elements.

based on a secure objective, is still outperformed by FIERSE. Fig. 5 shows the relation between worst-UE SLNR and the number of RIS elements. In all shown graphs, the minimum SLNR value increases with the increase of RIS elements. The reason for this is twofold. Firstly, as the number of RIS elements increases, the beams become narrower, hence focusing more precisely on the intended UE. This can decrease the leakage towards the eavesdropper and thus increase the SLNR value. Moreover, as the number of phase shifters on the RIS increases, the transmit power reflected toward the intended UE increases, and therefore the SLNR increases. Fig. 5 shows that maxmin-perfect and maxmin-statistical outperform both naive-maxmin and random-maxmin baselines. Since SSR-based approaches target overall secrecy rather than fairness, their minimum SLNR values are consistently low and not comparable to those of the max-min schemes. To maintain a clear and meaningful visualization, Fig. 5 includes only the max-min results, while the fairness behavior of SSR-based methods is later analyzed separately using Jain's index.

E. Impact of number of faulty elements

Fig. 6 plots the SSR versus the number of faulty reflecting elements. As expected, the no-fault curve remains constant with an increasing number of faulty elements, as this scheme assumes all elements operate ideally. Similarly, the random-maxmin graph is also flat since every phase shifter is randomly selected regardless of the faults in the RIS. For FIERSE, naive, AN, maxmin-perfect, and maxmin-statistical schemes, the SSR decreases steadily as additional elements fail. Even when about 86% of RIS elements are faulty ($B = 70$), FIERSE still offers the highest SSR (after no-fault), although the margin to the no-

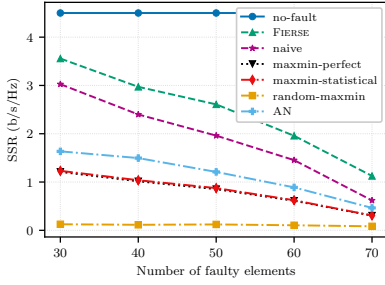


Fig. 6: SSR vs. number of faulty elements.

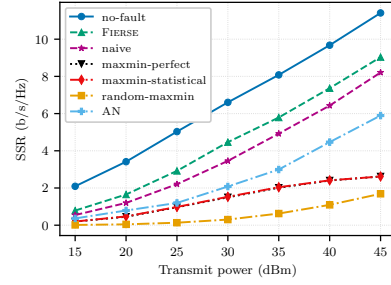


Fig. 8: SSR vs. transmit power.

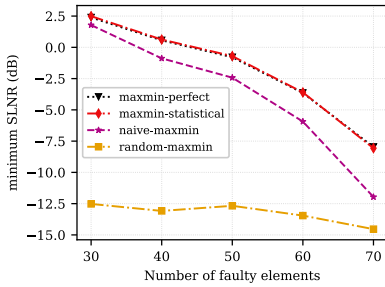


Fig. 7: SLNR vs. number of faulty elements.

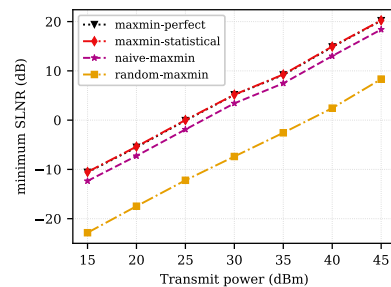


Fig. 9: Minimum SLNR vs. transmit power.

fault upper bound widens with increased faults. The smaller performance gap at high fault ratios results from saturation, where the overall RIS contribution becomes limited and all schemes converge toward the fault-dominated regime.

Fig. 7 shows the worst-UE SLNR as the number of faulty RIS elements increases. The maxmin-statistical, maxmin-perfect, and naive-maxmin schemes all experience a drop in SLNR as faults increase, but both maxmin-perfect and maxmin-statistical schemes retain the best minimum value. In contrast, the naive-maxmin variant, which ignores faults, drops sharply (from approximately 2 dB at 30 faults to approximately -12 dB at 70 faults), underperforming the max-min schemes by 3-4 dB as the number of faults increases. The random-maxmin baseline remains the worst as this scheme treats all RIS elements with random phase shifters; therefore, the corresponding graph exhibits random behavior.

These trends demonstrate that explicitly modeling RIS faults during optimization is crucial for maintaining robust beamforming and preventing information leakage. Even partial fault information prevents severe SLNR degradation compared to naive designs that ignore faulty elements.

F. Impact of transmit power on SSR and minimum SLNR

Fig. 8 depicts the SSR performance versus different transmit power. The SSR of all the schemes grows monotonically with P because a higher transmit power enhances the desired signal, thus increasing legitimate UE's SINR value over the eavesdropper. FIERSE follows the no-fault curve most closely achieving an SSR of approximately 8.5 b/s/Hz at $P = 45$ dBm, which outperforms naive, AN, and random-maxmin baselines. Both proposed max-min schemes outperform the random-maxmin, although underperform FIERSE. Furthermore, the gap

between FIERSE and the max-min schemes widens with P , confirming the efficiency of the SSR approach in high signal-to-noise ratio (SNR) regimes. The lower SSR of the max-min schemes relative to the naive baseline is expected. This arises because naive baseline maximizes SSR and ignores fairness across UEs, whereas max-min designs optimize the worst-UE SLNR. Their advantage becomes evident in Fig. 9, where both maxmin-perfect and maxmin-statistical outperform the naive-maxmin and random-maxmin baselines by about 3 dB and 12 dB, respectively, when $P = 45$ dBm. In this plot, the maxmin-perfect and maxmin-statistical curves are overlapping across the entire range, indicating that the statistical-CSI variant reproduces the behavior of the perfect-CSI design with negligible loss. Finally, the random-maxmin baseline shows the worst performance, highlighting the necessity of RIS configuration in enabling the communication.

G. Impact of number of UEs

Fig. 10 depicts the SSR performance versus different number of UEs for $B = 50$ and $N = 81$. As K grows, the max-min schemes see a mild SSR drop because power and RIS array gain are split across more streams and the inter-user interference increases which in turn enlarges the leakage power seen by the eavesdropper. In contrast, FIERSE slightly improves by exploiting multi-user diversity and tolerating inter-user leakage that also harms the eavesdropper. These trends highlight the distinct design objectives of the two methods: FIERSE prioritizes maximizing SSR, whereas the max-min schemes trade some SSR to improve the performance of the weakest UE. In addition, the behavior of the AN baseline illustrates the limitations of AN-based approaches in multi-UE

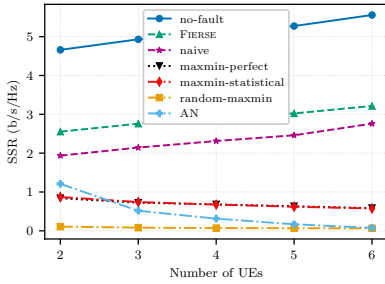


Fig. 10: SSR vs. number of UEs.

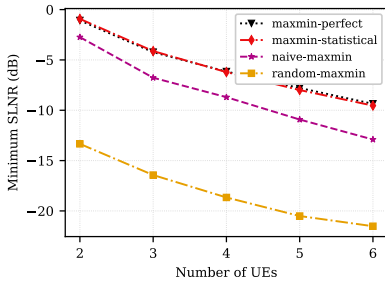


Fig. 11: Minimum SLNR vs. number of UEs.

scenarios. While AN attempts to degrade the eavesdropper by injecting interference into its channel, its effectiveness strongly depends on the availability of spatial degrees of freedom. As the number of UEs increases, the transmit power and beam-forming dimensions must be shared among more UEs, leaving fewer degrees of freedom to generate AN without harming legitimate receivers. Consequently, a significant portion of the injected noise leaks into the desired UE signals, which reduces the achievable SSR. This explains the monotonic SSR degradation observed for the AN curve as K grows and highlights why AN underperforms the proposed FIERSE framework as well as the maxmin-perfect and maxmin-statistical schemes in dense multi-UE scenarios.

Fig. 11 plots the worst-UE SLNR versus K . As depicted, all curves decrease with K , but both max-min designs deliver the highest minima. The statistical variant closely tracks the perfect variant, indicating robustness to channel uncertainty. Neglecting faults incurs a clear (and widening) gap as K increases, and the random-maxmin baseline is approximately 15–20 dB worse than the proposed schemes, as the random phase shifts prevent constructive control of the channels.

H. Impact of alternative fault models

To further evaluate the robustness of the proposed frameworks with respect to the adopted fault model, we performed a sensitivity analysis using alternative distributions for the faulty RIS amplitudes. In addition to the *uniform* model, we examined two additional fault models: (i) *Rayleigh*: a Rayleigh-distributed amplitude with random phase shifters where the scale parameter is set to $\sigma = 0.5$ and the generated amplitudes are truncated to ensure $\delta_b < 1$, $\forall B \in \{1, \dots, B\}$,

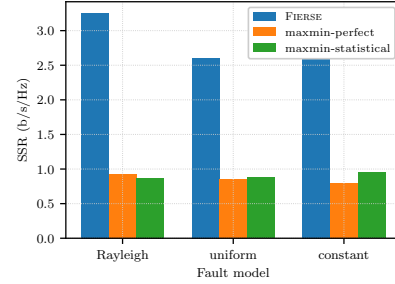


Fig. 12: SSR vs. fault models.

and (ii) *constant*: a constant attenuated amplitude with random phase shifters where the attenuation factor is $\delta_0 = 0.2$. Fig. 12 reports the resulting SSR obtained by the proposed FIERSE framework and the max-min SLNR schemes under these alternative models.

The results show that the qualitative behavior of all schemes remains consistent across the considered fault models. In particular, FIERSE consistently achieves the highest SSR, while the maxmin-based approaches provide lower but comparable performance. Importantly, we observe that the default fault model adopted throughout the paper, where the amplitude is uniformly distributed as $\delta_b \sim \mathcal{U}(0,1)$ along with a random phase shifter, produces the lowest SSR among the considered fault cases for each proposed framework. In contrast, the Rayleigh and constant models lead to slightly higher SSR values. This indicates that the original uniform model used in the paper represents a conservative scenario, where the faulty RIS elements behave as completely uncontrollable scatterers with arbitrary amplitude and phase responses. Therefore, the performance results reported in the paper can be interpreted as corresponding to a worst-case behavior among the considered fault models.

Overall, these results demonstrate that the conclusions of the paper remain robust under different faulty-element amplitude distributions, and that the adopted model provides a conservative characterization of RIS hardware faults.

I. Secrecy outage probability (SOP) vs secrecy threshold

Fig. 13 shows the SOP of the maxmin-perfect and maxmin-statistical schemes as a function of the secrecy rate threshold R_{th} for different numbers of faulty RIS elements. The SOP is defined using the minimum instantaneous UE secrecy rate in each channel realization. The SOP represents the probability that the achieved minimum secrecy rate falls below the target threshold. As expected, increasing the number of faulty RIS elements shifts the SOP curves to the left, indicating a higher likelihood of secrecy outage. For instance, when $B = 50$ faulty elements are present, the outage probability remains relatively low for moderate secrecy thresholds, while for $B = 70$ the outage probability increases significantly even for small thresholds. Overall, the results indicate that the proposed maxmin schemes maintain acceptable SOP performance under moderate hardware faults, while more severe faults progressively degrade reliability.

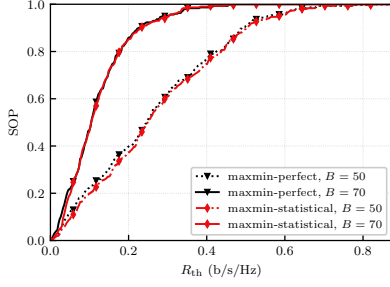


Fig. 13: SOP vs. secrecy rate threshold R_{th} (b/s/Hz).

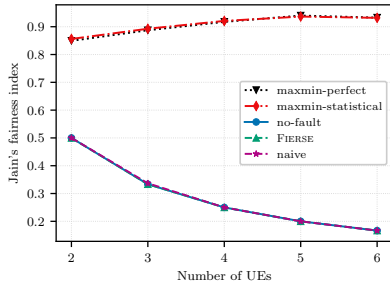


Fig. 14: Jain's fairness index vs. number of legitimate UEs.

J. Tradeoff: FIERSE vs. max-min SLNR

Fig. 14 reports Jain's fairness index computed from the per-UE secrecy rate vector $\mathbf{r}_s \triangleq [R_{s,1}, \dots, R_{s,K}]^T$ as

$$J(\mathbf{r}_s) = \frac{(\sum_{k \in \mathcal{K}} R_{s,k})^2}{K \sum_{k \in \mathcal{K}} R_{s,k}^2} \in [0, 1], \quad (57)$$

averaged over all Monte Carlo iterations. Values closer to 1 indicate ideal uniform secrecy rates across UEs. According to Fig. 14, maxmin-perfect and maxmin-statistical schemes maintain high fairness across different numbers of UEs, with $J \approx 0.86 - 0.94$ and only mild variation. In contrast, the no-fault, FIERSE, and naive approaches (all SSR-based) exhibit a monotonic descent starting from $J \approx 0.5$ at $K = 2$ to $J \approx 0.17$ at $K = 6$. This numerical study highlights the different roles played by the two optimization objectives pursued in this work. FIERSE maximizes the network SSR, whereas maxmin-perfect and maxmin-statistical maximize the worst-UE SLNR under perfect and statistical CSI, respectively. Both metrics control leakage created by faulty elements, but their computational profiles differ: an SLNR constraint is a single quadratic ratio that admits a tight SDR, so each bisection step reduces to solving only one interior-point SDP, initialized from a sufficiently small value of β^p or β^s . By contrast, FIERSE involves log-difference secrecy terms that require an inner SCA loop within each SDP. In practice, FIERSE yields the highest overall SSR but at higher complexity, whereas max-min SLNR trades an acceptable performance loss for fairness and lower complexity, which is preferable for fairness-critical deployments.

V. CONCLUSION

In this paper, we studied secure downlink MU-MISO with faulty RIS elements. We proposed FIERSE, an AO framework that jointly designs the BS precoder and functional RIS elements for SSR maximization, and introduced max-min SLNR variants under perfect and statistical CSI to trade peak SSR for fairness and lower complexity. The nonconvex problems are handled via quadratic transforms and MM/SCA within the AO loop. The key findings can be summarized as follows:

- **Fault modeling is essential.** Treating a faulty RIS as fully functional causes side-lobe leakage and secrecy loss. Our results demonstrate that FIERSE effectively compensates for these impairments, and consistently outperforms both naive and random baselines.
- **Fairness is tunable.** Max-min SLNR designs raise worst-UE SLNR with modest SSR loss and reduced complexity.
- **Resource scaling improves secrecy only when paired with fault-aware optimization.** Larger RIS or increased power improve secrecy when they are coupled with a fault-aware design. Otherwise, leakage can increase. Moreover, we showed that the statistical-CSI variant remains as effective as the perfect-CSI variant with only limited fault information.

Future work will study designs under statistical CSI for the eavesdropper and develop fairness-enhanced extensions of FIERSE.

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