Supplementary Material to "Performance Analysis of NSUM Estimators in Social-Network Topologies"

1 NSUM estimators

1.1 Basic estimators

The first frequentist estimators are characterized by using a combination of two procedures: an estimation of the network size and a posterior estimation of the unknown population. In particular, there are two different strategies for each procedure, consisting of using Eq. (1) or using a maximum likelihood approach. The estimation is sometimes replaced by a direct question about the size of the contact network as in (Garcia-Agundez et al., 2021).

Prior to deep into the estimators, we will set additional notation. We will denote as N_k the size of the population k, and p_k as the frequency of the population k in the total population, N_k/N.

First, we will focus on the estimate of the unknown subpopulation. It can be done if we have the information about the personal network size, d_i, but often this is not possible. (Killworth et al., 1998a) suggested using Eq. (1) to obtain n estimates of the number of the unknown population scaling by N the quotient of the report of the unknown subpopulation, y_{iu}, by the degree (d_i) of the individual i

$$N_u^i = N\frac{y_{iu}}{d_i}, \tag{S.1}$$

and then take the average to obtain an estimation of the unknown subpopulation size

$$\hat{N}_u^A = \frac{1}{n}\sum_{i=1}^{n} N_u^i. \tag{S.2}$$

The second proposal for the estimation of the unknown subpopulation was also introduced in (Killworth et al., 1998a). They assumed that the $y_{iu} \sim Bin(d_i, p_u)$ and they are independent, where p_u is N_u/N. Subsequently, they use a maximum likelihood estimate to obtain an estimation of N_u,

$$\hat{N}_u^{ML} = \frac{\sum_{i=1}^{n} y_{iu}}{\sum_{i=1}^{n} d_i}. \tag{S.3}$$

For the estimation of the personal network size, the process follows a similar process. One version employed the fundamental Eq. (1) and the known population to obtain L estimates of the d_i, and then take the average:

$$\hat{d}_i^A = \frac{1}{L}\sum_{k=1}^{L} d_i^k, \tag{S.4}$$

where the d_i^k is the estimate corresponding to $y_{ik}p_k^{-1}$. Note that the $N_k's$ are known. The second approach is similar to the case of the population estimates, and assumes that the y_{ik} are L independent random variables that follow a binomial distribution, $y_{ik} \sim Bin(d_i, p_k)$. The maximum likelihood is:

$$\hat{d}_i^{ML} = N \cdot \frac{\sum_{k=1}^{L} y_{ik}}{\sum_{k=1}^{L} N_k} \tag{S.5}$$

Combining these equations we lead to several plug-in estimators. The PiMLE estimator was the first NSUM estimator, it was introduced in (Killworth et al., 1998a) and combines Eq. (S.4) and Eq. (S.3):

$$\hat{N}_u^{PiMLE} = \frac{N}{n}\sum_{i=1}^{n} \frac{y_{iu}}{\hat{d}_i^A}. \tag{S.6}$$

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Another plug-in estimator is the MLE, which estimates the personal network size in a different manner using Eq. (S.5):

$$\hat{N}_u^{MLE} = N\frac{\sum_{i=1}^{n} y_{iu}}{\sum_{i=1}^{n} d_i^{ML}} = \sum_{i=1}^{n} y_{iu}\frac{\sum_{k=1}^{L} N_k}{\sum_{i=1}^{n}\sum_{k=1}^{L} y_{ik}}. \tag{S.7}$$

The MLE is by far the most popular estimator (see (Ocagli et al., 2021), for example) and its first appearance was in (Killworth et al., 1998b). The mean of sums estimator (MoS) is an alternative method proposed in (Killworth et al., 1998a) but owes its name to the article (Habecker et al., 2015). This method uses Eq. (S.4) and Eq. (S.2), and hence, it does not need the binomial assumptions.

$$\hat{N}_u^{MoS} = \frac{N}{n}\sum_{i=1}^{n} \frac{y_{iu}}{\hat{d}_i^A}, \tag{S.8}$$

The last combination of the previous equations is the one that uses Eq. (S.5) and Eq. (S.2). This estimator has not been employed yet, as (Kunke et al., 2024) point out, but we

consider it appropriate to include it since it can help to analyze the effect of the different combinations. The expression is:

$$\hat{N}_u^{KLN} = \frac{N}{n} \sum_{i=1}^{n} \frac{y_{iu}}{\hat{d}_i^{ML}} \tag{S.9}$$

On the other hand, some estimators can be produced by combining the equations of the population size, Eq. (S.2) and Eq. (S.3), with the values of the personal network size provided by the participants (δ_i). This is the approach of the study (Garcia-Agundez et al., 2021) when the elicitation of the network size is integrated with Eq. (S.3):

$$\hat{N}_u^{RoA} = N \frac{\sum_{i=1}^{n} y_{iu}}{\sum_{i=1}^{n} \delta_i}. \tag{S.10}$$

We employ another alternative using instead the means, the Average of Ratios (AoR, see Eq. (S.2)):

$$\hat{N}_u^{AoR} = \frac{N}{n} \sum_{i=1}^{n} \frac{y_{iu}}{\delta_i} \tag{S.11}$$

The different types of methods are summarized in the next table.

Degree \ Size	Average	Max. Likelihood
Average	MoS	KLN
Max. Likelihood	PiMLE	MLE
Direct	AoR	RoA

Table S.1: Basic NSUM estimators. The rows represent the way to obtain the degree and the columns the size

1.2 Generalized Network Scale-Up Method

The Generalized Network Scale-up Method (GNSUM) was suggested by (Feehan, Salganik, 2016) to reduce the transmission error. The approach is not based on Eq. (1) and includes additional data from the unknown subpopulation. This data is known as enriched aggregated relational data (EARD) and is made up of questions about the number of alters within an alter group that knows the participant's belonging to the unknown subpopulation. The alter groups are known subpopulations, that can differ from the known subpopulations

used to collect ARD. The EARD is used instead of the ARD of the known subpopulations. Additionally, the GNSUM has other assumptions such as the number of in-reports (number of people that a participant reports) is equal to the number of out-reports (number of participants that report an individual), and other conditions related to the visibility of the members of the unknown subpopulation within the alters groups (see (Feehan, Salganik, 2016) for more details). The GNSUM estimator is:

$$\hat{N}_u^{GNSUM} = \frac{\sum_{i \in s_F} y_{iu}/\pi_i}{\hat{\bar{v}}_{u,F}}, \tag{S.12}$$

where s_F is a sample from the frame population, π_i are the probability of inclusion from a sampling design, and $\hat{\bar{v}}_{u,F}$ is the estimated relative visibility defined as follows:

$$\hat{\bar{v}}_{u,F} = \frac{N_F}{N_{\mathcal{A}}} \frac{\sum_{i \in s_U} \sum_j \tilde{v}_{i,\mathcal{A}_j}/c\pi_i}{\sum_{i \in s_U} 1/(c\pi_i)}, \tag{S.13}$$

where s_U is a sample form the unknown subpopulation, $\mathcal{A}_j$ is the alter group j, $N_{\mathcal{A}}$ is the sum of the sizes of the alter groups, N_F the number of individuals of the frame population, $\tilde{v}_{i,\mathcal{A}_j}$ is the number of people in the alter group who knows that participant i belongs to the unknown population, and c a constant of proportionality.

In our particular case, we assume that the subset of possible participants is the entire population, and hence $N_F = N$. In addition, we use as alters groups the known subpopulations employed for degree estimation. Then, we have that $N_{\mathcal{A}} = \sum_k N_k$ and $\tilde{v}_{i,\mathcal{A}_j}$ is the number of people of the subpopulation j that are aware of the belonging of i to the unknown subpopulation, which without transmission error matches the response y_{ij}. Finally, we sample uniformly from the total population and the unknown subpopulations, and therefore, the inclusion probabilities are n/N and $|s_h|/N_u$.

The name of GNSUM comes from the theorem in (Feehan, Salganik, 2016), which says that under the conditions above, the GNSUM estimator is equal to the unknown subpopulation estimate Eq. (S.3) corrected by some adjustment factors (Feehan, Salganik, 2016). One of these adjustment factors is the transmission rate, τ (the ratio between the numbers of in-reports and the total connection from the unknown subpopulation to the rest of the frame). Additionally, (Feehan, Salganik, 2016) proposed using the correction of the MLE estimator. This is also known as GNSUM in the literature.

1.3 Overdispersed estimator

The Overdispersed estimator (Zheng et al., 2006) is the first Bayesian NSUM estimator to appear in the literature. It was created to handle the overdispersion of the data, which is likely produced by the variation of the degree distribution and the barrier error. The model is:

$$y_{ik} \sim Negative\ Binomial\left(mean = e^{\alpha_i+\beta_k}, overdispersion = \omega_k\right). \tag{S.14}$$

where $k = 1, \cdots, L$ or h, e^{α_i} is the expected degree of i, e^{β_k} is the proportion of links of the group k The Poisson distribution is due to the variation of the degrees in some real networks and the gamma distribution to model the barrier error. The ω_k represents a shape parameter of the gamma distribution of the propensity of forming ties with the subpopulation k. Moreover, they assign normal prior distribution for α_i and β_k, a uniform prior in the interval $(0, 1)$ to $1/\omega_k$, and proposed a Gibbs sampling. For further details about the prior distribution and the algorithm, see (Zheng et al., 2006).

1.4 Maltiel's Estimators (Maltiel et al., 2015)

A decade later, (Maltiel et al., 2015) proposed a Bayesian estimator and 4 modifications to handle the barrier error, the transmission error, the recall error, and combinations of them. The Maltiel et al. estimators include priors whose support is bounded. Whenever the proposed MCMC values are outside the range of the priors, a new value is proposed until a value falls in the prior range. Basto et al. pointed out that issue, they found that the estimators are sensitive to the variance of the network.

We found that, under our setting, the estimators do not get a reasonable acceptance rate. More concretely, we tested the estimators in our four main scenarios (see Table 2) in 20 samples, and we found that the estimators lasted more than two hours because of the acceptance rate, and some did not finish in three hours. In the two examples that finished, neither estimator was the best. Later, we tested the four estimators separately, and three of them lasted around 20-30 minutes in two scenarios, which is more than expected without acceptance rate issues. The rest of the combination lasted more than an hour. Additionally, in only one example, an estimator was the best. For that reason, we do not include these

estimators in the rest of the paper.

1.5 TPC Estimator (Teo et al., 2019)

(Teo et al., 2019) presented a model considering that the ARD follows a Poisson distribution. Let λ be a scale-up parameter such that $y_{ik} = \lambda N_k$, for every $k = 1, \cdots, L$ or h; and α_i is a parameter that characterizes the variability in the network size. The basic model of (Teo et al., 2019) is:

$$y_{ik} \sim Poisson\left(\lambda \alpha_i N_j\right). \tag{S.15}$$

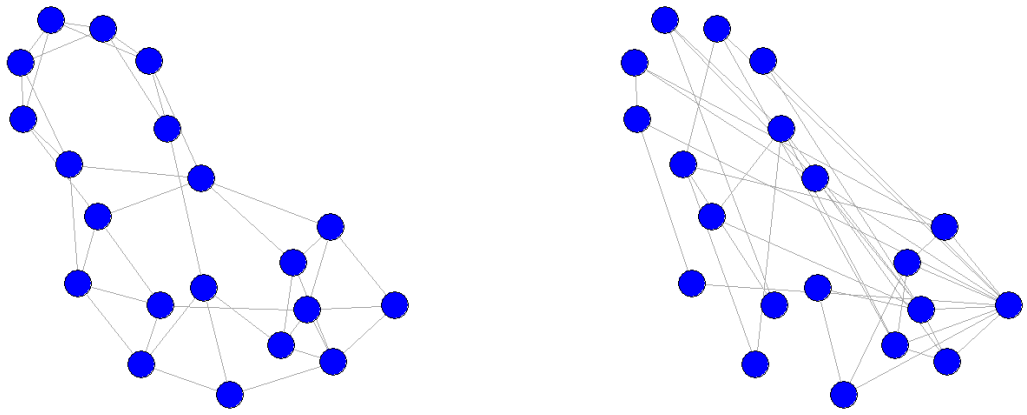
The prior distributions of the N_k are uniforms with large support to make them uninformative, and the prior of τ and λ are uniforms between 0 and 10. We include the algorithm of the supplementary material of (Zheng et al., 2006).

This work includes other model improvements, which allow transmission and barrier errors to be controlled. They introduce questions about the perceptions of the subpopulations and correction through demographic variables. These last adjustments are not included because they exceed the scope of the study.

2 Networks

Our interest lies in topologies with real properties. We use a small-world network (a Watts-Strogatz model) because many social networks present this type of effect. On the other hand, we use a scale-free network generated by a preferential attachment process, to evaluate the effect of the degree distribution since the other network selected has nodes with similar degrees.

Watts-Strogatz As a structure for social networks, the Watts–Strogatz approach is one the best options, since it has two important small-world properties: a high clustering coefficient and a short average path length (Newman, 2018). However, it has the disadvantage of its degree distribution, since it has a pronounced peak and then decays exponentially, obtaining a network where many nodes have similar degrees. The two variables of interest in the Watts–Strogatz model are the number of neighbors that each individual



(a) A Watts-Strogatz network (b) A preferential attachment network

Figure 1: Two networks with similar average degree. Figure (a) has more triadic closures and uniform degree distribution while Figure (b) has nodes with many links and few triangles.

has and the amount of randomness that is introduced to the connections between neighbors. The network is built as follows: the nodes are placed and are connected to the nearest neighbors. Then, with probability p (the probability of randomizing connections) an edge is removed and placed between two nodes chosen at random.

Preferential attachment The degree distribution of the Watts-Strogatz model is often unrealistic and some networks have scale-free structures with a power law degree distribution, which may represent better the degrees of a real social network such as the links in the World Wide Web and the citation of scientific papers (Newman, 2018). To model this possibility, we will consider the case of a Preferential Attachment. The construction of the graph is done by adding m vertices successively so that the vertices join proportionally to the node with the most connections. However, its clustering coefficient is small because it does not generate either local clustering or triadic closures.

According to the type of the network, the distribution of the unknown subpopulation may differ substantially. The infection stage in a SIR process on a network with a high clustering coefficient, such as the Watts-Strogatz, is more likely to stop or not reach all the nodes. In a network with few triadic closures opposition is hardly found (Easley et al., 2012). Furthermore, to compare the two approaches, we select the parameters of the SIR process so that the number of infected nodes is the same.

3 Recall error modeling

Modeling the personal network size recalling. Let δ_i be the personal network size. We want that $y_{iu} \leq d_i$ to get consistent responses. Additionally, we want the truncated normal to be symmetric and with mean δ_i. For that reason, the simulated degree is generated by rounding a two-sided truncated normal distribution with mean $\mu = \delta_i$, standard deviation $\sigma_i = \delta_i \rho$, lower bound $a = y_{iu} - 0.5$ and upper bound $b = 2\delta_i - y_{iu} + 0.5$:

$$TN(\mu = \delta_i, \sigma = \delta_i \rho, a = y_{iu} - 0.5, b = 2\delta_i - y_{iu} + 0.5). \tag{S.16}$$

The lower limit is selected because the declared degree, δ_i, must be greater than the ARD of the unknown subpopulation. The upper limit comes from the fact that if $y_{iu} \leq \delta_i$ then $\delta_i \leq 2\delta_i - y_{iu}$. We add and subtract 0.5 so that the boundaries of the normal can be reached. In addition, the mean and bounds are also selected to have the expected value of the real personal size.

Modeling the ARD. Similarly, we obtain the ARD with barrier error, y_{ik}, by rounding a truncated normal:

$$TN(\mu = x_{ik}, \sigma = x_{ik}\rho, a = z_{ik} - 0.5, b = 2x_{ik} - z_{ik} + 0.5) \tag{S.17}$$

where x_{ik} is the real number of people to whom individual i is connected, z_{ik} is the real number of contacts that belong to both the unknown subpopulation and subpopulation k, and $\rho \in (0, 1)$ is the memory factor. We assume that the recall error is also present in the question associated with the unknown subpopulation. There are two natural bound: $y_{iu} \geq 0$ and $y_{iu} \leq \delta_i$. But, we add some constraints to get a distribution as symmetrical as

possible, $a = max(-0.5, 2x_{iu} - \delta_i + 0.5)$ and $b = min(2x_{iu} + 0.5, \delta_i + 0.5$. The truncated normal distribution is then:

$$TN(\mu = x_{iu}, \sigma = x_{iu}\rho, a, b). \tag{S.18}$$

The differences of the intervals from the mean are equal to $min(x_{iu}, \delta_i - x_{iu})$. Later, we get the y_{iu} value taking a value of the distribution and rounding it

Modeling the EARD. The philosophy in this case is the same. For simplicity, we do not consider the possible combination of barrier error in the other variables as we did before. We model the visibility report $\tilde{v}_{i,\mathcal{A}_k}$, by rounding the truncated normal

$$N(\mu = v_{i,\mathcal{A}_k}, \sigma = v_{i,\mathcal{A}_k}\rho, a = -0.5, b = 2v_{i,\mathcal{A}_k} + 0.5), \tag{S.19}$$

where $v_{i,\mathcal{A}_k}$ is the total number of people who know they belong to the unknown subpopulation of i.

4 Results

We show the results of the simulation. The results are presented in tables with rows representing the values for the mean, standard deviation, MAE, and MSE for the different NSUM estimators, and columns representing the parameters of the simulation. The W-U scenario represents the case of a Watts-Strogatz network and an unknown population distributed uniformly. The W-S scenario is the case of a Watts-Strogatz network and SIR process for the unknown population. The case of the preferential attachment network and uniform distribution of the unknown population is the P-U scenario. Finally, the P-S represents the preferential attachment network and SIR process.

For the transmission error simulation, we show the results for the W-U scenario in Tables S.2 and S.3, the W-S scenario in Tables S.4 and S.5, the P-U scenario in Tables S.6 and S.7, and the P-S scenario in Tables S.8 and S.9.

Regarding the recall error simulations, we provide the results for the W-U scenario in Table S.10, the W-S scenario in Table S.11, the P-U scenario in Table S.12, and the P-S scenario in Table S.13.

In the case of the number of subpopulations, we show the results for the W-U scenario in Table S.14, the W-S scenario in Table S.18, the P-U scenario in Table S.22, and the P-S scenario in Table S.26. Additionally, we include all the cases of disjoint subpopulations in Tables S.15, S.19, S.23 and S.27, all the cases of not disjoint small subpopulations in Tables S.16, S.20, S.24, S.28, and the cases of disjoint and small subpopulations in Tables S.17, S.21, S.25 and S.29.

For the subpopulation size simulations, we include the results of the W-U scenario in Table S.30, the W-S scenario in Table S.32, the P-U scenario in Table S.34, and the P-S scenario S.36. The cases of disjoint subpopulations are in Tables S.31, S.33, S.35 and S.37.

Finally, for the sample size, we show the W-U scenario in Table S.38, the W-S scenario in Table S.39, the P-U scenario in Table S.40, and the P-S scenario in Table S.41.

Metric	Method	Visibility factor									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Mean	PiMLE	12.01	22.66	28.2	41.62	50.76	59.8	64.92	80.82	85.92	100.86
	MLE	12.19	23.05	28.2	40.69	50.3	59.22	63.57	78.59	84.35	100.18
	MoS	12.43	23.51	28.65	42.75	52.47	61.07	66.05	82.36	88.56	103.15
	KLN	12.23	23.13	28.37	40.84	50.37	59.37	63.73	78.57	84.38	100.43
	GNSUM	132.13	**106.21**	**90.41**	**103.83**	**96.76**	**97.49**	**91.53**	**99.51**	**92.23**	96.2
	Overd.	**99.01**	147.55	147.53	177.69	192.22	181.36	184.49	187.54	204.86	239.67
	TPC	16.37	27.35	32.31	45	54.17	63.35	67.55	82.29	88.26	103.54
	RoA	12.24	22.95	28.2	40.99	50.39	59.22	63.89	78.36	84.32	100.13
	AoR	12.03	22.85	28	41.19	50.13	59.24	64.18	78.6	84.68	**99.88**
SD	PiMLE	**6.67**	**8.83**	11.98	15.33	10.98	**12.41**	17.09	18.69	19.87	17.77
	MLE	6.9	9.31	12.02	14.76	**10.92**	12.52	15.78	18.58	18.28	17.49
	MoS	7	9.12	12.03	16.53	11.11	13.14	17.86	19.55	21.7	17.91
	KLN	6.98	9.44	12.2	14.91	11.14	12.78	**15.75**	18.69	18.33	17.68
	GNSUM	76.2	50.99	36.54	39.59	21.71	21.63	22.66	23.81	21.27	**16.09**
	Overd.	40.95	69.03	73.77	111.31	96.95	106.57	137.67	104.84	117.47	160.47
	TPC	6.9	9.52	12.11	**14.61**	10.92	12.87	15.77	18.24	**17.85**	17.98
	RoA	6.95	9.26	12.01	14.99	11.35	12.61	16.29	17.88	18.7	17.05
	AoR	6.94	9.16	**11.79**	15.09	11.14	12.79	16.47	**17.81**	18.75	16.89
MAE	PiMLE	87.99	77.34	71.8	58.38	49.24	40.2	35.74	21.63	20.7	13.64
	MLE	87.81	76.95	71.8	59.31	49.7	40.78	36.66	23.15	20.14	13.57
	MoS	87.57	76.49	71.35	57.25	47.53	38.93	34.65	21.22	21.09	13.8
	KLN	87.77	76.87	71.63	59.16	49.63	40.63	36.41	23	20.15	13.56
	GNSUM	62.85	**38.22**	**31.92**	**33.34**	**17.8**	**17.17**	**19.89**	**20.1**	19.7	13.7
	Overd.	**31.07**	63.62	61.52	99.06	101.21	91.56	107.01	100.36	112.21	142.9
	TPC	83.63	72.65	67.69	55	45.83	36.65	33.08	20.5	**17.73**	13.56
	RoA	87.76	77.05	71.8	59.01	49.61	40.78	36.62	23.12	20.53	13.2
	AoR	87.97	77.15	72	58.81	49.87	40.76	36.37	22.73	20.66	**13.12**
MSE	PiMLE	7783.72	6055.46	5292.1	3631.6	2539.05	1762.83	1508.16	699.75	573.13	300.68
	MLE	7755.99	6004.27	5292	3724.59	2583.59	1812.2	1564	786.07	562.31	290.65
	MoS	7715.37	5929.26	5228.2	3537.13	2376.27	1679.66	1456.08	674.16	578.26	314.63
	KLN	7749.46	5993.17	5271.79	3710.75	2580.88	1806.18	1551.25	791.13	563.33	297.05
	GNSUM	6549.21	**2508.96**	**1360.62**	**1503.47**	**458.19**	**450.91**	**559.6**	**538.94**	490.14	**260.45**
	Overd.	**1593.92**	6788.51	7429.02	17805.1	17434.88	17409.63	25144.66	18105.89	24102.97	43971.43
	TPC	7038.37	5364.51	4720.88	3228.01	2213.91	1500.85	1289	629.61	**440.41**	319.64
	RoA	7747.38	6017.36	5291.61	3695.85	2583.41	1814.32	1555.79	771.96	578.08	276.26
	AoR	7785.17	6031.17	5315.98	3675.19	2604.54	1816.62	1540.67	759.3	568.63	270.95

Table S.2: Metrics for the simulations varying the visibility factor in the W-U scenario.

Metric	Method	Visibility factor									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Mean	PiMLE(adj)	121.74	102.58	104.24	102.63	105.24	103.92	**100.16**	112.66	102.56	103.26
	MLE(adj)	117.76	102.25	99.85	100.44	103.93	103.08	98.14	109.83	100.59	103.77
	MoS(adj)	123.73	105.53	104.54	104.54	105.78	106.17	102.92	114.3	102.98	105.6
	KLN(adj)	117.96	102.76	99.41	**100.24**	103.61	103.04	98.7	109.48	**99.96**	103.79
	GNSUM	**114.57**	93.16	97.72	95.06	**100.34**	**100.89**	98.4	**109.32**	97.01	**99.88**
	Overd.(adj)	1039.99	649.97	453.57	471.88	356.59	261.88	231.5	228.07	231.38	187.98
	TPC(adj)	161.28	121.61	112.62	110.57	111.93	110.42	104.2	115.05	105.43	108.11
	RoA(adj)	117.69	101.56	99.69	100.29	103.83	103.04	98.14	109.49	101.17	102.76
	AoR(adj)	115.35	**101.1**	**100.02**	100.72	103.93	103.35	98.21	109.76	101.48	102.78
SD	PiMLE(adj)	82.9	35.9	40.08	35.04	27.49	20.38	23.1	21.33	17.6	23.11
	MLE(adj)	80.28	37.43	39.76	31.97	27.51	19.65	22.81	20.42	**17.2**	22.95
	MoS(adj)	84.87	35.89	40.46	35.91	30.16	20.28	23.04	23.43	17.91	23.73
	KLN(adj)	79.94	37.33	39.35	31.97	28.43	19.63	22.78	20.95	17.43	23.36
	GNSUM	**72.9**	**35.65**	41.61	**30.97**	**26.25**	**18.45**	22.85	**19.41**	18.43	**22.41**
	Overd.(adj)	505.65	191.56	190.42	246.84	231.07	154.63	151.65	158.24	128.25	103.1
	TPC(adj)	83.33	36.8	40.13	32.48	28.63	20.35	22.78	20.22	18.04	23.73
	RoA(adj)	80.2	36.38	38.98	32.33	27.01	19.22	22.53	20.5	17.77	23.01
	AoR(adj)	79.89	35.71	**38.51**	32.68	26.93	19.21	**22.4**	19.86	18.18	23.09
MAE	PiMLE(adj)	67.01	28.86	34.07	27.85	18.24	16.62	16.99	20.44	14.81	19.08
	MLE(adj)	65.4	28.85	34.4	26.36	18.73	16.09	16.32	18.65	**14.01**	**18.72**
	MoS(adj)	70.31	29.2	33.93	29.02	20.39	17.49	17.33	22.13	14.97	19.63
	KLN(adj)	65.71	28.89	34.05	26.45	19.11	16.39	16.8	18.89	14.01	19.03
	GNSUM	**59.13**	28.27	36.3	**23.36**	**18.02**	**15.35**	16.28	**17.78**	14.51	18.84
	Overd.(adj)	939.99	549.97	359.18	371.88	260.84	164.41	137.57	133.62	137.74	93.14
	TPC(adj)	73.57	33.01	**32.71**	27.69	21.14	17.7	16.39	21.5	14.99	20.21
	RoA(adj)	65.14	27.46	33.39	26.43	18.84	15.94	**15.94**	18.34	14.4	18.87
	AoR(adj)	64.44	**27.28**	33.15	26.55	18.73	16.21	16.21	18.24	14.7	18.88
MSE	PiMLE(adj)	7001.48	1231.35	1544.1	1173.48	745.37	409.98	506.93	592.63	300.99	518.21
	MLE(adj)	6438.46	1335.67	1502.19	971.22	734.35	376.28	497.87	492.71	**281.4**	514.71
	MoS(adj)	7406.61	1254.22	1575.87	1245.84	897.66	428.92	512.78	725.9	313.74	566.37
	KLN(adj)	6394.01	1331.69	1471.41	971.01	780.92	375.47	494.5	506.66	288.69	532.68
	GNSUM	**5261.27**	1253.93	1649.83	**935.49**	**654.54**	**324.32**	498.4	**444.7**	331.52	**476.98**
	Overd.(adj)	1126481.97	337321.22	159459.1	196183.38	116565.05	48922.41	39140.37	40190.13	32885.62	17839.67
	TPC(adj)	10352.2	1753.25	1689.07	1114.18	921.15	502.07	510.82	614.61	338.64	600.91
	RoA(adj)	6422.9	1260.03	1443.29	992.81	707.46	360.24	485.55	489.1	301.5	510.39
	AoR(adj)	6299.2	**1212.54**	**1408.9**	1014.83	704.2	361.63	**479.88**	470.03	316.15	514.09

Table S.3: Metrics for the simulations of corrected estimators varying visibility factor in the W-U scenario.

Metric	Method	Visibility factor									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Mean	PiMLE	8.78	16.47	30.91	34.23	41.88	55.02	51.29	78.2	93.43	93.8
	MLE	8.79	16.14	30.99	34.51	42.29	57.47	51.62	81.18	93.3	91.65
	MoS	9.05	16.37	30.43	33.76	42.8	54.2	52.08	76.95	90.83	92.59
	KLN	8.8	16.32	30.95	34.56	42.31	57.41	51.75	81.42	92.78	92.19
	GNSUM	94.39	**78**	**100.41**	**84.91**	**83.06**	**92.5**	**71.4**	**99.62**	103.07	88.38
	Overd.	**95.9**	122.66	131.56	156.02	163.42	170.8	161.55	174.34	239.82	165.36
	TPC	12.94	20.08	34.81	38.94	46.23	60.93	55.47	85.23	**97.67**	**95.68**
	RoA	8.82	16.14	30.85	34.22	41.92	57.71	51.95	81.11	93.12	91.5
	AoR	8.96	16.14	31.15	34.31	41.87	57.58	52.38	80.71	93.96	93.02
SD	PiMLE	**7.14**	12.96	17.29	17.78	18.59	28.16	27.67	35.9	40.53	55.22
	MLE	7.15	12.8	18.43	19.04	18.4	29.59	28.04	38.88	39.48	51.82
	MoS	7.37	13.28	**16.54**	**16.82**	18.7	**25.83**	**27.38**	**34.19**	**36.07**	52.04
	KLN	7.2	13.02	18.41	18.93	18.73	29.01	28.08	38.81	38.8	52.24
	GNSUM	85.38	65.28	55.45	49.75	38.11	49.02	37.05	48.08	45.54	**49.69**
	Overd.	37.43	57.05	57.52	80.81	56.68	91.48	62.81	75.47	96.82	102.25
	TPC	7.17	12.77	18.18	19.07	18.41	29.77	28.22	38.92	39.41	52.05
	RoA	7.21	**12.75**	18.13	18.96	**18.3**	29.81	28.44	38.83	39.13	52.22
	AoR	7.27	12.85	18.51	18.97	18.5	30.64	29.48	38.12	40.38	54.96
MAE	PiMLE	91.22	83.53	69.09	65.77	58.12	49.24	52.7	34.8	33.51	41.39
	MLE	91.21	83.86	69.01	65.49	57.71	47.74	52.32	35.49	33.35	41.22
	MoS	90.95	83.63	69.57	66.24	57.2	48.83	51.77	34.62	**30.43**	39.9
	KLN	91.2	83.68	69.05	65.44	57.69	47.45	52.19	35.47	32.7	41.04
	GNSUM	66.14	59.97	**38.64**	**41.42**	**34.68**	**35.67**	**40.67**	37.7	36.72	40.92
	Overd.	**29.41**	**43.19**	58.97	62.34	69.47	76.73	66.05	83.36	139.82	81.13
	TPC	87.06	79.92	65.19	61.06	53.77	44.84	49.44	**33.39**	32.9	**39.84**
	RoA	91.18	83.86	69.15	65.78	58.08	47.63	52.4	35.45	33.2	41.86
	AoR	91.04	83.86	68.85	65.69	58.13	48.09	52.68	34.99	33.99	43.06
MSE	PiMLE	8368.61	7136.11	5056.86	4626.12	3705.95	2776.79	3100.18	1699.5	1604.13	2935.13
	MLE	8368.01	7187.69	5084.66	4633.63	3652.05	2640.39	3087.48	1790.34	1525.41	2620.42
	MoS	8324.05	7161.44	5100.32	4656.47	3604.44	2732.02	3008.29	**1641.52**	**1319.8**	2627.75
	KLN	8367.46	7164.1	5089.9	4622.94	3661.93	2613.37	3077.31	1776	1481.98	2653.7
	GNSUM	6956.14	4532.48	**2921.25**	**2579.42**	**1666.85**	**2339.16**	**2121.79**	2196.23	1979.48	**2480.87**
	Overd.	**1347.51**	**3605.58**	4138.63	9341.81	7073.85	12962.25	7535.83	10936.5	28455.36	14204.42
	TPC	7627.62	6542.63	4563.63	4073.33	3213.71	2368.45	2739.63	1657.32	1480.58	2592.68
	RoA	8364.1	7186.31	5093.41	4668.73	3691.84	2632.52	3077.27	1789.14	1502.25	2663.11
	AoR	8338.17	7188.9	5065.4	4656.49	3704.28	2691.65	3093.25	1752.84	1585.47	2918.28

Table S.4: Metrics for the simulations varying visibility factor in the W-S scenario.

Metric	Method	Visibility factor									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Mean	PiMLE(adj)	126.32	90.18	108.05	87.08	94.29	90.56	88.24	91.71	96.29	102.42
	MLE(adj)	125.93	89.19	111.62	89.2	94.13	94.5	89	93.39	94.59	101.19
	MoS(adj)	**121.35**	88.74	**106.06**	87.23	93.68	87.16	87.37	87.6	96.22	100.91
	KLN(adj)	125.47	89.11	110.76	89.57	94.09	94.52	89.13	92.9	94.56	101.61
	GNSUM	126.72	84.19	108.68	88.13	91.87	92.78	86.47	89.24	91.15	98.95
	Overd.(adj)	997.4	590.16	400.69	341.07	297.03	262.19	233.79	195.8	233.27	177.6
	TPC(adj)	165.36	**108.88**	125.47	**98.96**	**101.04**	**101.18**	**95.17**	**98.07**	**99.65**	104.78
	RoA(adj)	127.29	88.69	111.56	88.58	95.22	94.07	88.67	92.93	94.71	**100.38**
	AoR(adj)	130	90.12	111.49	88.19	94.84	94.38	88.55	93.58	95.4	101.9
SD	PiMLE(adj)	96.33	66.61	59.91	42.41	56.1	54.87	42.96	48.81	46.75	67.75
	MLE(adj)	96.51	65.66	63.63	42.49	56.14	61.96	45.26	47.61	45.7	66.72
	MoS(adj)	**90.77**	63.67	**58.02**	42.38	**53.77**	**50.5**	**41.02**	**43.72**	43.85	**62.24**
	KLN(adj)	95.62	65.33	62.68	42.99	55.13	61.65	44.92	47.44	44.89	67.1
	GNSUM	94.61	**61.9**	64.78	41.94	55.88	59.38	44.99	44.42	**43.14**	65.37
	Overd.(adj)	402.6	288.1	237.32	131.31	95.51	98.25	109.25	100.34	96.31	76.06
	TPC(adj)	95.03	64.94	63.61	43.01	55.74	62.19	45.59	47.6	46.02	65.91
	RoA(adj)	98.11	65.33	63.74	**41.88**	57.25	61.55	45.4	46.8	45.84	65.92
	AoR(adj)	99.86	66.55	64.44	42.55	55.82	61.84	45.3	47.95	47.12	67.74
MAE	PiMLE(adj)	75.19	56.88	49.36	34.43	39.6	44.74	35.98	39.54	36.08	50.63
	MLE(adj)	74.87	56.94	53.18	36.12	39.52	48.17	37.85	38.75	36.72	51.02
	MoS(adj)	**71.79**	54.86	**47.96**	**34.17**	37.8	**41.45**	**34.9**	37.29	**33.4**	**47.16**
	KLN(adj)	74.83	56.67	52.27	36.55	38.96	47.71	37.73	38.59	36.36	51.1
	GNSUM	73.82	**53.49**	53.67	35.35	40.82	47.15	37.79	**36.5**	35.38	49.69
	Overd.(adj)	897.4	491.01	300.69	241.15	197.03	162.69	141.74	111.79	133.27	82.51
	TPC(adj)	86.23	55.94	54.67	35.19	**36.43**	47.02	36.84	37.62	35.73	49.32
	RoA(adj)	75.96	56.92	53.56	35.32	40.27	48.14	37.48	38.37	36.24	50.38
	AoR(adj)	77.08	57.87	53.86	35.64	39.88	48.01	37.36	39.2	36.55	51.53
MSE	PiMLE(adj)	9509.11	4311.46	3474.81	1875.86	3022.32	2949.56	1891.34	2332.5	2090.3	4366.45
	MLE(adj)	9519.82	4212.42	3981.78	1831.58	3029.02	3676.96	2067.35	2196.68	2013.63	4230.35
	MoS(adj)	**8283.02**	3978.12	**3235.08**	1869.74	**2786.71**	**2587.54**	**1757.73**	**1969.74**	**1840.69**	**3681.12**
	KLN(adj)	9334.4	4172.88	3847.89	1864.14	2922.72	3640.13	2035.07	2188.3	1943.77	4279.76
	GNSUM	9216.49	**3890.11**	4062.49	1812.07	3032.95	3401.77	2106	1989.91	1846.03	4061.25
	Overd.(adj)	959318.68	319113.12	143915.57	74494.95	47487.66	35475.9	29239.74	18742.15	26573.18	11517.79
	TPC (adj)	12851.75	4085.75	4492.67	**1758.39**	2952.53	3675.23	1997.74	2156.58	2011.64	4150.21
	RoA(adj)	9888.97	4182.06	3993.66	1796.82	3136.05	3633.62	2086.1	2131.05	2024.14	4128.28
	AoR(adj)	10374.12	4304.73	4076.96	1859.66	2987.22	3664.77	2080.84	2225.18	2130.08	4363.18

Table S.5: Metrics of the corrected methods and the GNSUM for the simulations varying the visibility factor in the W-S scenario.

Metric	Method	Visibility factor									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Mean	PiMLE	8.08	22.09	28.75	36.15	46.55	60.55	67.94	91.13	92.92	98.35
	MLE	8.71	21.36	25.84	38.54	46.85	58.45	64.01	84.96	88.91	92.28
	MoS	8.48	21.97	29.51	36.14	47.4	62.26	68.77	**94.15**	92.42	**99.38**
	KLN	8.67	21.39	25.62	38.41	46.4	58.14	63.72	85.12	87.92	91.69
	GNSUM	**116.27**	**114.59**	**100.29**	**100.02**	**94.16**	**118.18**	**103.75**	109.75	**101.9**	102.17
	Overd.	72.56	129.54	120.53	155.84	203.47	172.12	135.88	166.18	167.89	182.92
	TPC	12.86	25.86	29.69	42.74	51.45	62.71	67.79	89.23	92.97	96.81
	RoA	8.75	21.35	25.44	38.3	46.77	58.51	63.69	84.3	87.63	91.96
	AoR	8.1	21.45	27.14	35.86	46.95	58.54	66.31	85.61	89.99	96.52
SD	PiMLE	6.96	11.33	9.86	14.09	14.91	21.41	23.14	23.47	25.88	16.06
	MLE	7.05	**8.99**	8.6	12.72	**14.84**	**16.89**	18.69	20.61	20.63	11.86
	MoS	7.19	11.91	10.97	13.39	16.79	22.42	24.6	24.3	25.04	15.5
	KLN	7.01	9.04	8.58	**12.48**	15.16	16.92	**18.26**	20.81	**19.98**	**11.72**
	GNSUM	99.8	49.79	39.34	37.16	33.82	45.1	31.04	29.9	42.03	33.56
	Overd.	39.36	44.44	73.62	59.26	123.24	126.99	98.07	124.19	84.52	112.6
	TPC	**6.74**	9.29	8.7	12.71	15.53	17.17	18.42	21.03	20.52	11.79
	RoA	7.07	9.02	**8.31**	12.9	14.88	17.02	18.59	**20.48**	20.09	12.3
	AoR	7	11.01	9.06	13.44	15.06	19.9	22.54	21.66	22.19	15.93
MAE	PiMLE	91.92	77.91	71.25	63.85	53.45	41.42	34.04	21.15	22.63	12.46
	MLE	91.29	78.64	74.16	61.46	53.15	41.87	37.28	21.08	20.15	10.19
	MoS	91.52	78.03	70.49	63.86	52.6	40.21	34.19	21.21	21.95	11.43
	KLN	91.33	78.61	74.38	61.59	53.6	42.17	37.29	21.27	20.05	10.77
	GNSUM	82.34	**41.41**	**30.01**	**30.13**	**25.85**	**31.25**	**26.43**	23.14	29.58	23.46
	Overd.	**40.91**	45.06	59.92	62.64	118.86	95.73	62.66	82.68	74.25	90.99
	TPC	87.14	74.14	70.31	57.26	48.55	38.22	33.75	**19.24**	**18.7**	**8.76**
	RoA	91.25	78.65	74.56	61.7	53.23	41.71	37.57	21.66	20.04	10.2
	AoR	91.9	78.55	72.86	64.14	53.05	42.6	35.03	21.37	20.8	11.56
MSE	PiMLE	8495.96	6191.22	5169.45	4265.08	3067.9	1992.2	1536.19	602.06	686.48	247.84
	MLE	8380.33	6261.65	5570.12	3931.47	3033.84	1997.26	1627.54	629.6	527.37	193.24
	MoS	8424.62	6224.08	5083.61	4248.47	3034.99	1901.86	1550.55	595.28	653.23	228.56
	KLN	8387.08	6257.87	5602.71	3941.83	3091.71	2023.83	1633.04	633.09	525.11	199.59
	GNSUM	9726.38	**2567.96**	**1470.69**	**1311.96**	**1120.44**	2263.01	**929.41**	944.28	1681.48	1074.88
	Overd.	**2225.15**	2749.19	5571.06	6454.57	25135.87	20522.36	10423.45	19033.16	11395.52	18920.86
	TPC	7636.37	5578.04	5015.97	3432.7	2586.15	**1670.41**	1359.87	**536.17**	**449.44**	**142.17**
	RoA	8374.38	6263.4	5624.87	3964.62	3043.61	1996.67	1646.48	644.94	536.38	208.53
	AoR	8491.79	6285.39	5387.07	4285.43	3029.68	2095.32	1617.84	653.03	568.26	253.18

Table S.6: Metrics for the simulations varying the visibility factor in the P-U scenario.

Metric	Method	Visibility factor									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Mean	PiMLE(adj)	93.98	**101.15**	105.89	102.33	97.65	93.56	101.87	103.8	95.62	111.58
	MLE(adj)	85.51	96.28	102.93	104.68	88.47	88.21	97.39	97.16	93.06	102.16
	MoS(adj)	101.72	104.95	106.96	101.85	**98.64**	94.05	103.8	103.92	**97.63**	112.49
	KLN(adj)	84.64	95.74	102.17	104.15	88.16	88.07	97.28	96.94	92.54	101.44
	GNSUM	**101.31**	105.58	**100.71**	115.94	96.15	89.36	**99.05**	98.73	107.06	117.82
	Overd.(adj)	900.18	584.27	437.98	456.85	385.11	228.96	184.22	245.01	177.04	165.46
	TPC(adj)	124.85	117.14	116.98	115.14	97.15	**95.51**	103.37	102.53	97.27	105.96
	RoA(adj)	85.81	95.74	103.14	104.27	87.66	88.1	97.05	95.94	93.06	**101.25**
	AoR(adj)	90.25	97.84	105.99	**99.71**	93.25	90.68	98.58	**99.08**	93.89	109.09
SD	PiMLE(adj)	86.67	52.78	57.14	38.29	29.68	25.82	19.4	28.97	24.17	23.3
	MLE(adj)	64.94	37.8	38.96	31.32	18.7	22.97	**18.43**	25.22	21.18	17.83
	MoS(adj)	97.11	55.23	60.17	38.44	30.56	27.04	18.66	28.94	25.52	23.61
	KLN(adj)	**64.01**	37.44	38.8	31.23	18.6	23	18.71	25.66	**21.15**	17.62
	GNSUM	87.17	60.75	45.15	54.94	46.09	32.57	23.65	41.92	39.5	34.73
	Overd.(adj)	442.37	252.31	238.52	222.76	158.23	128.4	101.12	176.73	111.12	115.97
	TPC(adj)	66.61	**37.19**	**37.62**	**30.59**	19.08	**22.57**	18.85	25.14	21.3	**17.43**
	RoA(adj)	65.66	37.43	39.73	31.61	**17.82**	23.05	18.47	**24.39**	21.28	17.85
	AoR(adj)	81.25	43.73	52.26	34.28	25.4	24.14	18.98	27.74	22.18	22.93
MAE	PiMLE(adj)	70.05	36.4	42.4	29.37	22.51	21.1	15.56	23.8	19.48	21.48
	MLE(adj)	56.09	**30.84**	29.03	27.66	17.71	20.13	15.39	21.28	16.95	15.54
	MoS(adj)	77.83	37.23	44.2	29.2	22.77	22.58	14.69	24.13	20.52	22.19
	KLN(adj)	**55.24**	30.94	**28.96**	27.48	17.52	20.3	15.68	21.52	16.92	15.26
	GNSUM	70.29	35.83	34.09	45.59	30.32	27.58	18.35	32.03	34.09	28.99
	Overd.(adj)	800.18	484.27	337.98	356.85	287.24	137.5	85.83	165.74	81.12	66.9
	TPC(adj)	59.98	31.51	29.88	29.6	**15.82**	**19.29**	**14.57**	**19.63**	**16.86**	**15.17**
	RoA(adj)	56.81	30.92	29.52	27.92	17.79	19.88	15.61	20.99	17.09	15.57
	AoR(adj)	66.64	32.86	39.56	**26.89**	21.81	20.26	16.24	23.77	17.71	20.79
MSE	PiMLE(adj)	7171.76	2647.74	3136.62	1398.4	842.57	674.6	361.05	811.49	574.13	649.73
	MLE(adj)	4216.48	1371.2	1450.7	953.63	465.34	640.27	**329.6**	612.47	474.21	306.68
	MoS(adj)	8962.58	2922.01	3487.91	1407.12	889.28	729.86	345.37	811.21	624.23	685.79
	KLN(adj)	**4128.45**	1349.53	**1434.73**	**943.58**	469.1	644.67	339.87	634.78	480.55	**297.1**
	GNSUM	7220.54	3536.79	1937.52	3121.74	2033.34	1120.85	532.25	1670.68	1532.04	1463.7
	Overd.(adj)	826190.35	294991.18	168276.37	174480.94	105073.22	32293.57	16808.22	50699.31	17666.11	17061.03
	TPC(adj)	4832.8	1608	1632.7	1118.21	**354.1**	**504.17**	348.82	606.98	**438.59**	324.25
	RoA(adj)	4297.45	**1348.82**	1509.14	967.17	454.12	646.3	332.78	**581.78**	478.19	304.26
	AoR(adj)	6366.9	1821.73	2630.89	1116.72	658.44	640.46	344.42	732.08	504.76	582.11

Table S.7: Metrics of the corrected methods and the GNSUM for the simulations varying the visibility factor in the P-U scenario.

Metric	Method	Visibility factor									
	PiMLE	14.77	36.22	58.63	74.32	92.75	113.4	134.39	154.43	163.96	194.51
	MLE	15.09	36.23	56.82	71.81	91.79	106.49	130.48	143.51	156.38	183.16
	MoS	15.28	37.38	57.65	75.02	93.18	116.13	135.84	157.96	165.19	198.56
	KLN	15	36.36	56.24	71.57	91.09	105.83	129.66	143.06	155.35	182.03
Mean	GNSUM	91.99	**109.21**	**123.24**	**118.62**	120.68	122.12	**117.06**	**127.17**	**93.18**	**116.84**
	Overd.	**92.56**	134.36	140.66	181.32	183.77	204.49	177.63	231.67	176.63	192.52
	TPC	19.23	40.55	60.69	76.53	**96.06**	110.23	134.47	147.64	160.16	186.94
	RoA	15.16	36.27	56.44	71.56	91.89	**105.68**	130.63	142.11	156.63	183.34
	AoR	14.18	34.95	55.42	71.2	89.55	109.24	129.47	146.42	158.96	189.1
	PiMLE	5.18	12.84	20.25	17.54	22.09	24.77	26.04	29.18	42.07	34.68
	MLE	5.59	12.64	17.88	15.77	20.79	21.2	20.44	25.97	25.5	28.85
	MoS	5.98	14.34	18.7	17.61	21.68	24.99	25.26	31.37	41.99	36.39
	KLN	5.62	12.85	17.86	**15.32**	**20.45**	21.23	**19.71**	26.21	**25.38**	**28.09**
SD	GNSUM	47.34	55.79	70.55	69.01	45.57	41.19	53.92	51.12	34.87	54.82
	Overd.	47.8	94.96	96.31	90.27	105.16	139.95	110.77	161.39	54.59	69.62
	TPC	5.87	13.1	17.81	15.49	20.78	21.47	20.46	25.54	26.03	28.11
	RoA	5.71	12.57	**17.65**	15.72	20.45	**20.17**	20.53	**24.93**	25.83	28.67
	AoR	**4.66**	**11.88**	17.76	17.13	21.33	21.49	23.99	27.83	37.47	31.69
	PiMLE	85.23	63.78	42.3	27.03	18.88	23.88	34.64	54.43	63.96	94.51
	MLE	84.91	63.77	43.18	28.19	17.32	18.99	30.48	44	56.38	83.16
	MoS	84.72	62.62	42.43	26.65	18.5	25.42	36.6	57.96	65.19	98.56
	KLN	85	63.64	43.76	28.43	17.32	18.67	**29.66**	43.63	55.35	82.03
MAE	GNSUM	**37.09**	**45.25**	51.85	43.34	38.57	38.96	40.19	**38.99**	**26.37**	**36.6**
	Overd.	41.16	72.12	73.47	92.24	94.3	112.21	82.72	137.96	76.63	97.95
	TPC	80.77	59.45	**39.31**	**23.47**	**15.82**	20.39	34.47	47.64	60.16	86.94
	RoA	84.84	63.73	43.56	28.44	16.74	**17.57**	30.63	42.85	56.63	83.34
	AoR	85.82	65.05	44.58	30.46	19.12	19.93	29.8	46.54	58.96	89.1
	PiMLE	7290.52	4224.32	2100.73	951.63	515.91	762.6	1826.7	3771.4	5771.89	10075.53
	MLE	7238.94	4218.43	2168.42	1030.96	478.12	469.28	1326.06	2533.73	3795.9	7706.89
	MoS	7211.05	4116.06	2125.7	918.79	493.16	853.5	1890.42	4294.02	5925.03	10972.58
	KLN	7255.57	4207.13	2217.54	1031.12	476.6	462.31	**1249.15**	2506.43	3675.77	7477.94
MSE	GNSUM	**2192.91**	**3042.29**	5268.78	4870.51	2400.78	2101.29	3052.81	3220.45	**1201.47**	**3138.55**
	Overd.	2225.96	9747.4	10465.66	14354.12	17522.4	29523.73	17683.47	42081.16	8703.81	13165.08
	TPC	6556.88	3696.89	**1846.39**	**778.94**	**425.58**	542.58	1585.59	2888.96	4263.22	8309.44
	RoA	7228.33	4211.7	2193.77	1043.79	463.26	**418.83**	1338.69	**2363.62**	3840.93	7726.25
	AoR	7385.57	4365.81	2287.29	1108.26	541.51	523.87	1415.14	2891.11	4810.45	8892.46

Table S.8: Metrics for the simulations varying the visibility factor in the P-S scenario.

Metric	Method	Visibility factor									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Mean	PiMLE(adj)	222.25	167.33	182.87	175.43	188.12	189.88	194.61	182.76	185.48	193.19
	MLE(adj)	222.92	185.79	173.96	171.38	189.48	189.41	187.25	182.03	175.8	184.81
	MoS(adj)	223.18	165.58	185.35	177.67	191.03	193.34	195.96	188.97	188.36	197.15
	KLN(adj)	221.92	184.6	172.58	170.37	188.01	187.9	186.2	181.44	174.92	183.53
	GNSUM	**134.01**	**129.68**	**101.63**	**91.82**	**114.55**	**114.27**	**119.23**	**106.46**	**96.85**	**113.54**
	Overd.(adj)	1175.74	680.04	527.11	556.4	384.28	239.66	226.17	202.92	189.69	192.56
	TPC(adj)	262.04	205.51	186.49	181.73	197.28	196.63	192.96	188.1	180.66	188.75
	RoA(adj)	221.62	184.77	174.53	170.68	187.41	188.69	186.29	181.4	174.83	183.89
	AoR(adj)	218.62	169.76	178.38	174.46	180.92	184.19	188.65	175	179.99	187.68
SD	PiMLE(adj)	93.39	52.13	67.07	51.97	34.8	39.78	38.27	45.53	39.27	28.27
	MLE(adj)	81.12	50.15	48.37	53.88	33.19	31.08	36.94	54.32	31.99	27.24
	MoS(adj)	91.56	54.4	68.62	53.48	39.52	41.07	40.54	48.8	37.58	27.8
	KLN(adj)	80.63	**49.81**	48.18	53.67	33.58	**29.59**	36.72	55.26	30.88	**26.82**
	GNSUM	**71.29**	85.13	**42.95**	**32.09**	50.83	38.7	72.82	**43.26**	**30.53**	56.17
	Overd.(adj)	469.81	372.84	250.74	301.39	227.96	91.54	94.4	86.16	138.15	68.05
	TPC(adj)	86.44	49.84	46.38	54.66	32.41	32.25	**36.6**	54.9	32.04	28.4
	RoA(adj)	80.58	50.71	48.04	53.38	**29.9**	31.59	37.91	53.49	31.33	27.79
	AoR(adj)	85.93	53.03	66.59	52.92	30.61	37.11	39.34	43.6	36.2	27.39
MAE	PiMLE(adj)	129.28	72.05	86.03	75.86	88.12	91.48	94.61	84.19	85.48	93.19
	MLE(adj)	124.59	87.98	75.94	72.15	89.48	89.41	87.25	82.05	75.84	84.81
	MoS(adj)	130.64	70.31	88.94	78.52	91.03	94.54	95.96	89.82	88.36	97.15
	KLN(adj)	123.64	86.83	74.81	71.49	88.01	87.9	86.2	81.65	74.92	83.53
	GNSUM	**55.4**	**61.15**	**35.8**	**26.65**	**37.34**	**30.44**	**50.83**	**34.73**	**23.88**	**42.71**
	Overd.(adj)	1075.74	580.04	427.11	456.4	284.28	139.95	130.64	107.14	95.39	99.01
	TPC(adj)	162.04	106.05	86.74	81.73	97.28	96.63	92.96	88.1	80.66	88.75
	RoA(adj)	123.03	86.7	76.83	71.95	87.41	88.69	86.29	81.49	74.83	83.89
	AoR(adj)	125.16	73.51	80.86	74.46	80.92	85.6	88.65	77.22	79.99	87.68
MSE	PiMLE(adj)	23230.6	7115.56	11141.3	8255.88	8915.41	9581.93	10341.55	8817.9	8771.03	9442.98
	MLE(adj)	21361.26	9748.7	7693.13	7852.94	9052.73	8912.43	8908.35	9532.44	6717.18	7898.02
	MoS(adj)	23137.41	**7111.98**	11757.93	8749.39	9770.65	10315.81	10768.99	10178.59	9148.22	10171.22
	KLN(adj)	21041.67	9514.64	7472.95	7688.37	8816.04	8557.65	8712.17	9533.25	6519.57	7659.96
	GNSUM	**5985.7**	7766.23	**1755.1**	**1045.25**	**2666.23**	**1626.27**	**5406.92**	**1819.29**	**895.19**	**3180.56**
	Overd.(adj)	1366912.64	468504.15	242147.59	294590.76	130184.4	27467.63	24385.16	17644.71	26176.65	12965.57
	TPC(adj)	33353.8	13492.79	9524.71	9518.5	10461.19	10325.61	9915.01	10624.34	7482.25	8642.46
	RoA(adj)	20959.08	9628.85	7746.63	7702.82	8489.81	8814.26	8811.85	9343.17	6531.82	7770.86
	AoR(adj)	21086.72	7538.17	10356.34	8204.78	7437.25	8395.78	9328.68	7429.96	7644.19	8400.26

Table S.9: Metrics of the corrected methods and the GNSUM for the simulations varying the visibility factor in the P-S scenario.

Metric	Method	Memory factor									
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Mean	PiMLE	102.28	105.69	95.65	107.81	107.21	106.65	110.03	103.26	107.93	97.81
	MLE	100.68	103.65	94.57	104.41	102.58	101.42	103	100.16	**100.7**	92.28
	MoS	103.77	107.81	96.82	109.67	108.64	109.32	113.29	106.48	110.23	**100**
	KLN	**100.66**	103.2	94.72	104.37	103.1	101.29	103.68	**100.15**	101.85	92.49
	GNSUM	97.98	**100.38**	90.29	**101.51**	**99.72**	**98.95**	**100.47**	98.2	97.91	88.92
	Overd.	232.05	143.29	146.58	171.16	188.05	151.75	198.75	139.2	196.96	212.31
	TPC	104.19	107.41	98.74	109.03	106.99	105.58	106.75	104.19	105.13	96.17
	RoA	100.75	103.31	95.15	105.49	104.51	101.71	105.67	98.95	99.1	91.83
	AoR	101.24	104.07	**99.11**	117.57	155.79	152.75	229.46	212.23	176.12	205.27
SD	PiMLE	16.75	20.56	18.06	18.98	29.92	22.49	27.72	30.63	27.93	33.81
	MLE	15.77	20.9	16.79	18.09	**26.82**	20.49	25.49	30.35	24.78	31.46
	MoS	16.31	22.63	17.21	20.13	31.33	23.54	28.47	31.14	29.03	36.48
	KLN	**14.99**	21.42	16.41	**17.84**	27.47	20.65	25.76	30.23	25.38	31.71
	GNSUM	17.29	22.32	**16.11**	17.91	28.8	23.77	26.17	28.83	**23.31**	**28.41**
	Overd.	158.51	46.44	81.32	81.33	111.88	75.79	93.7	77.44	92.16	115.32
	TPC	15.28	**20.55**	16.71	18.45	27.2	20.15	**25.45**	30.45	24.6	31.68
	RoA	16.23	21.39	16.54	19.5	32.25	**19.81**	28.55	**28.41**	25.46	32.2
	AoR	16.22	20.79	19.67	22.21	101.62	69.69	109.37	81.86	82.1	124.24
MAE	PiMLE	14.29	16.37	13.69	16.13	22.76	18.91	21.78	23.96	20.11	27.89
	MLE	13.44	16	12.72	14.8	**20.21**	16.83	20.44	24.65	17.98	26.71
	MoS	13.56	18.12	13.25	17.99	23.44	19.93	23.31	25.4	21.37	29.91
	KLN	**12.69**	16.58	**12.5**	14.44	20.29	16.87	**20.26**	24.65	18.66	27.24
	GNSUM	14.32	17.26	15.17	**14.37**	22.68	20.37	21.07	**23.74**	**17.84**	**24.83**
	Overd.	135.71	47.51	55.36	78.09	96.28	63.35	101.22	67.87	100.98	124.29
	TPC	13.41	16.13	12.65	16.01	20.56	16.34	20.3	24.36	18.67	26.36
	RoA	13.68	16.3	12.95	14.97	22.31	**15.71**	23.51	24.04	18.05	27.49
	AoR	13.52	**15.98**	15.08	21.59	60.59	60.34	137.84	112.23	85.94	121.7
MSE	PiMLE	271.6	433.93	328.72	403.32	902.54	524.75	830.67	902.12	804.2	1091.07
	MLE	236.66	428.15	297.23	330.27	**689.84**	400.8	**626.46**	875.19	583.74	999.89
	MoS	266.86	547.33	291.37	478.63	1007.24	613.08	946.72	963.27	905.12	1264.18
	KLN	**213.87**	446.07	283.69	321.37	726.6	406.6	644.08	867.99	615.29	1011.68
	GNSUM	287.94	473.23	340.98	**306.97**	788.15	537.94	651.06	793.05	**520.67**	**889.35**
	Overd.	41306.32	3923.59	8452.12	11348.53	19644.51	8134.91	18093.17	7232.84	17471.49	25246.33
	TPC	239.4	456.27	**266.82**	404.94	751.76	417.01	660.84	898.34	601.01	967.82
	RoA	250.87	445.68	283.42	391.55	1008.68	**375.79**	806.43	**767.79**	616.71	1051.93
	AoR	251.52	**427.29**	368.4	777.49	12923.23	7395.55	28122.71	18961.32	12197.02	25745.87

Table S.10: Metrics for the simulations varying the memory factor in the W-U scenario.

Metric	Method	Memory factor									
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Mean	PiMLE	107.59	96.2	92.95	88.29	91.42	106.47	81.95	81.5	88.25	101.15
	MLE	109.55	96.11	93.19	88.86	88.22	102.96	81.02	79.34	84.68	95.92
	MoS	**104.87**	94.35	92.77	87	85.08	102.29	82.62	79.68	87.58	100.71
	KLN	109.57	96.83	93.49	89.31	87.27	103.26	80.74	78.64	84.69	95.93
	GNSUM	106.44	94.36	92.38	84.65	84.87	**101.64**	78.39	78.41	82.37	90.88
	Overd.	213.72	216.31	180.06	196.88	180.97	187.9	172.41	142.59	181.35	187.71
	TPC	114.26	**100.32**	97.27	93.18	92	106.6	84.72	**83.38**	**89.34**	**100.38**
	RoA	109.01	95.94	93.85	89.1	90.11	103.81	80.88	78.84	86.56	96.94
	AoR	109.4	98.52	**100.72**	**99.56**	**105.92**	160.04	**111.86**	129.03	130.5	156.82
SD	PiMLE	56.21	53.84	35.08	39.86	51.41	60.42	31.66	50.24	53.33	69.62
	MLE	58.11	53.5	38.45	42.48	47.3	60.48	30.83	47.53	51.97	62.19
	MoS	**51.37**	**49.75**	**34.87**	**38.35**	**42.18**	**56.02**	**28.51**	47.76	49.71	69.84
	KLN	57.96	53.88	38.66	42.98	46.03	60.57	30.01	**46.64**	50.75	62.1
	GNSUM	56.56	52.95	37.76	40.49	44.77	58.31	31.12	47.63	**49.62**	**59.17**
	Overd.	85.42	104.25	47.41	82.6	93.64	106.1	87.77	58.77	79.48	82.36
	TPC	58.19	53.65	38.7	42.61	46.89	60.12	30.86	46.96	52.38	61.76
	RoA	57.26	52.57	39.66	43.17	50.09	59.57	29.93	49.88	54.47	66.82
	AoR	58.18	57.84	42.26	47.22	56.47	87.13	63.16	89.09	85.55	91.03
MAE	PiMLE	44.06	41.91	27.41	35.24	44.19	41.11	29.38	46.45	46.3	50.98
	MLE	43.82	41.1	31.39	36.43	42.64	42.13	30.15	43.22	46.27	47.58
	MoS	**41.4**	**39.22**	**26.83**	**34.78**	**38.89**	**38.05**	**27**	44.56	**44.17**	52.73
	KLN	44.19	40.81	31.19	36.77	41.81	42.52	29.85	43.42	45.81	47.91
	GNSUM	43.07	41.24	30.76	36.49	41.44	41.4	31.97	44.68	45.75	**46.21**
	Overd.	114.77	122.44	81.16	100	90.79	96.53	82.14	54.48	86.1	96.74
	TPC	43.82	40.15	30.5	35.47	41.03	41.07	28.12	**40.14**	45.11	46.59
	RoA	43.39	40.67	31.76	36.73	44.11	41.12	30.17	43.54	46.47	47.91
	AoR	44.84	43.26	33.22	39.73	47.22	77.28	48.53	59.64	67.03	74.99
MSE	PiMLE	3059.59	2768.13	1218.77	1646.38	2584.74	3510.11	1277.93	2739.95	2839.9	4606.5
	MLE	3298.72	2734.17	1450.72	1838.82	2264.24	3483.47	1262.97	2572.54	2800.81	3690.78
	MoS	**2530.31**	**2383.69**	**1207.53**	**1566.37**	**1912.95**	**2986.1**	**1074.18**	2580.33	**2501.74**	4634.09
	KLN	3283.13	2768.33	1462.27	1869.48	2174.96	3496.44	1226.52	2523.12	2681.16	3680.15
	GNSUM	3080.61	2695.76	1412.81	1793.37	2132.94	3232.41	1386.86	2621.36	2650.18	**3409.47**
	Overd.	19864.88	23853.04	8545.33	15866.32	14886.89	18420.49	12561.81	5095.13	12619.57	14137.47
	TPC	3420.64	2734.93	1430.18	1771.48	2153.07	3476.83	1138.19	**2371.56**	2720.59	3623.19
	RoA	3195.38	2642.22	1531.81	1889.43	2481.17	3385.26	1216.39	2811.67	2998.97	4250.78
	AoR	3304.41	3179.88	1697.46	2118.46	3065	10818.05	3930.23	8383.37	7884.06	11099.51

Table S.11: Metrics for the simulations varying the memory factor in the W-S scenario.

Metric	Method	Memory factor									
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Mean	PiMLE	**99.07**	94.13	104.98	96.77	**99.65**	95.65	101.72	96.26	108.72	99.57
	MLE	94.26	92.91	99.55	94.03	90.08	86.93	94.2	92.9	96.35	91.4
	MoS	101.28	95.26	105.92	**99.64**	99.63	**97.08**	104.73	96.32	108.94	**100.09**
	KLN	93.75	91.8	98.91	93.18	89.71	86.43	93.89	91.33	96.18	90.94
	GNSUM	111.04	91.06	**100.45**	97.15	96.74	89.28	97.36	**98.56**	102.56	106.04
	Overd.	139.06	163.3	164.06	197.53	182.53	178.37	192.98	172.72	189.3	152.36
	TPC	98.57	**96.96**	104.12	97.82	94.3	91.03	**98.32**	97.18	**100.25**	95.11
	RoA	93.75	92.38	98.82	94.27	89.86	88.42	94.06	93.96	97.97	89.63
	AoR	95.22	92.77	104.04	102.98	113.09	128.67	118.76	145.05	184.94	160.49
SD	PiMLE	18.6	22.01	29.24	24.8	40	22.63	35.4	**26.28**	28.38	38.31
	MLE	18.02	19.08	24.86	19	29.15	**17.84**	30.54	27.21	19.28	28.5
	MoS	20.02	22.05	28.24	24.78	41.21	24.44	35.49	27.27	26.89	40.1
	KLN	17.99	19.09	24.6	**18.44**	**28.6**	18.31	30.34	26.45	**19.17**	**28.29**
	GNSUM	37.7	23.14	29.5	31.96	35.2	26.94	41.73	44.7	27.2	35.31
	Overd.	65.12	71.72	108.91	107.71	89.55	121.77	89.11	77.91	72.81	73.98
	TPC	18.55	18.97	25.29	19.06	29.3	18.18	**30.15**	27.35	19.18	28.82
	RoA	**17.35**	**18.38**	**24.36**	19.83	29.66	19.1	33.52	28.21	21.17	32.97
	AoR	17.99	22.11	27.26	26.92	47.25	48.04	67.03	54.63	113.99	75.57
MAE	PiMLE	**13.5**	16.85	22.35	17.39	33.15	19.43	27.22	**20.71**	23.88	32.52
	MLE	15.09	15.74	20.43	14.75	22.26	19.11	23.51	22.62	16.37	23.56
	MoS	15.17	16.92	21.8	16.28	33.5	20.79	27.22	22.31	22.27	34.17
	KLN	14.91	16.42	20.29	14.29	22.16	19.43	23.43	21.94	**16.16**	**23.11**
	GNSUM	27.64	19.59	23.36	27.54	28.04	22.26	31.24	33.61	22.29	27.12
	Overd.	47.48	69.55	71.36	107.12	89.89	85.26	98.06	83.31	96.41	64.68
	TPC	15.58	**14**	20.93	**14.23**	**21.71**	**16.85**	**22.89**	22.98	16.83	23.46
	RoA	14.78	15.81	**19.65**	15.57	22.13	19.68	26	24.81	18.16	26.12
	AoR	14.94	17.23	20.39	20.8	37.14	41.48	41.96	56.08	88.62	75.69
MSE	PiMLE	329.43	494.62	837.16	594.76	1519.8	505.28	1193.27	**670.32**	841.42	1394.51
	MLE	341.4	396.14	587.51	378.65	905.44	472.96	919.63	753.62	366.62	845.34
	MoS	382.55	484.24	792.76	583.25	1613.47	575.96	1218.64	720.19	766.67	1527.93
	KLN	346.53	413.63	576.29	369.58	882.69	502.73	912.04	739.68	363.61	842.19
	GNSUM	1472.32	588.55	826.66	978.27	1187.72	804.46	1661.08	1900	709.25	1220.89
	Overd.	5553.6	8894.56	15372.59	20533.75	14428.89	20229.21	16188.8	11053.87	13010.14	7941.5
	TPC	328.84	**351.18**	624.54	**349.8**	**848.34**	**394.25**	**866.54**	718.61	**349.42**	**812.89**
	RoA	**325**	379.07	**565.23**	406.55	938.55	480.77	1102.62	792.51	429.96	1140.17
	AoR	330.25	516.84	722.37	697.17	2292.08	3014.88	4620.57	4864.23	19557.81	9084.39

Table S.12: Metrics for the simulations varying the memory factor in the P-U scenario.

Metric	Method	Memory factor									
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Mean	PiMLE	200.38	198.41	183.87	184.16	193.84	208.99	184.54	188.93	189.92	188.69
	MLE	187.15	185.69	186.13	174.12	185.72	197.34	171.46	174.41	175.24	184.13
	MoS	202.8	199.59	184.88	187.99	196.93	208.08	186.25	192.41	195.26	190.46
	KLN	185.89	183.9	185.11	174.06	183.65	196.95	171.31	174.6	174.94	183.32
	GNSUM	**109.24**	**121.2**	**129.49**	**136.1**	**115.65**	**128.93**	**100.25**	**122.08**	**102.15**	**123.11**
	Overd.	229.88	206.57	217	184.08	206.29	192.64	227.49	215.51	260.4	204.75
	TPC	191.6	189.37	190.6	178.35	189.88	201.9	174.87	178.93	179.26	188.33
	RoA	185.56	183.07	188.18	170.78	187.95	192.97	173.99	170.05	182.05	190.14
	AoR	191.11	190.96	185.13	200.12	257.22	294.86	300.35	254.59	288.47	333.94
SD	PiMLE	33.47	38.28	**33.87**	37.58	39.59	42.05	47.69	51.37	47.27	41.86
	MLE	20.47	32.15	53.92	24.54	40.93	45.17	34.38	**32.39**	33.46	39.21
	MoS	33.67	38.04	35.98	35.8	40.08	40.79	50.84	55.13	54.46	39.6
	KLN	**19.94**	31.23	52.72	23.36	**39.49**	46.7	**33.59**	32.86	**32.38**	**38.42**
	GNSUM	34.8	41.15	61.82	52.35	52.31	54.35	44.61	61.2	48.52	45.22
	Overd.	113.38	117.31	77.75	72.58	100.27	101.82	107.3	136.07	196.55	102.75
	TPC	19.95	32.29	54.19	24.67	40.86	45.22	34.5	32.72	33.37	38.89
	RoA	20.46	**30.92**	58.64	**22.25**	42.87	**37.49**	40.88	32.93	43.98	42.16
	AoR	29.4	33.82	39.18	39.72	62.23	112.49	149.06	72.38	85.67	125.79
MAE	PiMLE	100.38	98.41	83.87	84.16	93.84	108.99	84.54	88.93	89.92	88.69
	MLE	87.15	85.69	86.13	74.12	85.72	97.34	71.46	74.41	75.24	84.89
	MoS	102.8	99.59	84.88	87.99	96.93	108.08	86.42	92.41	95.26	90.46
	KLN	85.89	83.9	85.11	74.06	83.65	96.95	71.31	74.6	74.94	84.24
	GNSUM	**27.19**	**31.26**	**42.79**	**45.95**	**36.87**	**43.86**	**34.74**	**50.51**	**34.64**	**37.72**
	Overd.	129.88	112.66	117	88.14	112.16	101.36	127.49	118.71	164.6	108.87
	TPC	91.6	89.37	90.6	78.35	89.88	101.9	74.87	78.93	79.26	88.73
	RoA	85.56	83.07	88.18	70.78	87.95	92.97	73.99	70.05	82.05	90.14
	AoR	91.11	90.96	85.13	100.12	157.22	194.86	200.35	154.59	188.47	233.94
MSE	PiMLE	11141.16	11076.29	8123.6	8424.8	10295.77	13558.79	9308.43	10416.34	10209.22	9530.3
	MLE	7993.08	8325.01	10180.22	6065.39	8939.76	11413.36	6229.23	6533.19	6724.9	8538.23
	MoS	11645.57	11292.13	8435.47	8959.41	10921.32	13262.55	9895.1	11425.99	11892.85	9673.24
	KLN	7754.68	7964.86	9883.93	6003.38	8478.96	11471.06	6156.87	6590.54	6612.12	8344.43
	GNSUM	**1235.84**	**2058.41**	**4500.54**	**3906.6**	**2844.92**	**3642.67**	**1890.61**	**4045.58**	**2240.97**	**2476.53**
	Overd.	29080.73	24430.35	19431.68	12073.31	20849.67	18431.26	27192.45	30930.54	62427.75	21002.41
	TPC	8768.21	8978.24	10998	6717.58	9664.68	12326.71	6736.47	7246.4	7340.97	9238.49
	RoA	7718.12	7809.68	11041.69	5480.37	9480.66	9979.78	7061.55	5936.99	8569.7	9814.44
	AoR	9122.75	9359.96	8704.49	11522.81	28395.65	49990.85	61250.35	28875.36	42491.26	69759.95

Table S.13: Metrics for the simulations varying the recall error in the P-S scenario.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	**90.6**	102.61	105.76	104.67	**99.47**	**99.65**	100.94	105.03	113.2	101.21
	MLE	85.28	**99.9**	100.98	**101.51**	96.92	97.75	101.05	104.4	111.9	100.28
	MoS	**90.6**	102.61	105.76	104.67	**99.47**	**99.65**	100.94	105.03	113.2	101.21
	KLN	85.28	**99.9**	100.98	**101.51**	96.92	97.75	101.05	104.4	111.9	100.28
	GNSUM	84.01	100.56	103.55	102.92	97.67	101.13	100.99	**103.94**	**110.39**	100.69
	Overd.	215.07	390.79	474.38	495.08	676.35	713.95	748.42	782.52	854.07	828.62
	TPC	89.49	103.96	105.03	105.86	100.65	102.18	104.73	108.13	116.31	104.21
	RoA	86.45	99.86	100.69	101.83	96.8	98.1	**100.89**	104.25	111.76	100.31
	AoR	86.22	99.89	**100.48**	102.07	97.51	98.34	100.91	104.14	111.88	**100.24**
SD	PiMLE	20.01	23.98	**24.16**	21.87	24.01	23.82	20.19	27.82	24.95	20.35
	MLE	18.81	22.93	24.33	21.2	23.17	22.59	19.81	28.49	24.87	**19.47**
	MoS	20.01	23.98	**24.16**	21.87	24.01	23.82	20.19	27.82	24.95	20.35
	KLN	18.81	22.93	24.33	21.2	23.17	22.59	19.81	28.49	24.87	**19.47**
	GNSUM	**18.16**	23.15	25.78	21.64	23.62	23.09	19.45	28.67	25.89	20.74
	Overd.	93.98	282.71	268.05	334.58	296.69	297.42	280.53	255.07	109.29	232.91
	TPC	19.01	23.18	24.91	21.27	23.25	22.75	**19.32**	28.39	24.91	19.85
	RoA	19.64	22.89	24.79	20.65	**22.83**	**22.42**	19.94	27.92	25.26	20.01
	AoR	19.32	**22.71**	24.75	**20.26**	22.87	22.59	20.09	**27.49**	**24.79**	20.34
MAE	PiMLE	**18.4**	18.09	**20.39**	17.61	17.83	19.9	17.36	**21.69**	17.26	15.2
	MLE	20.67	18.05	20.91	16.13	18.38	18.38	17.06	22.15	17.06	**14.81**
	MoS	**18.4**	18.09	**20.39**	17.61	17.83	19.9	17.36	**21.69**	17.26	15.2
	KLN	20.67	18.05	20.91	16.13	18.38	18.38	17.06	22.15	17.06	**14.81**
	GNSUM	19.82	18.02	21.64	16.85	18.05	18.62	**15.96**	22.78	17.86	15.17
	Overd.	116.37	292.32	374.38	397.64	576.35	613.95	648.42	682.52	754.07	728.62
	TPC	18.79	18.36	21.94	17.23	**17.1**	18.18	17.26	22.22	19.32	14.99
	RoA	20.93	18.11	20.91	15.62	18.07	**18.18**	16.86	21.84	17.64	15.44
	AoR	20.66	**17.83**	20.56	**15.52**	17.86	18.34	17.01	21.79	**17.01**	15.81
MSE	PiMLE	468.69	553.06	587.68	476.17	548.07	539.04	388.29	760.74	765.69	394.91
	MLE	552.75	499.72	**563.24**	429.06	519.31	490	373.86	790.64	729.31	**360.34**
	MoS	468.69	553.06	587.68	476.17	548.07	539.04	388.29	760.74	765.69	394.91
	KLN	552.75	499.72	**563.24**	429.06	519.31	490	373.86	790.64	729.31	**360.34**
	GNSUM	568.89	509.56	644.1	453.55	535.58	507.8	**360.2**	796.36	744.94	409.15
	Overd.	21632.08	160485.18	208418.28	262436.21	415805.29	460965.91	495212.28	527645.6	579972.56	582421.19
	TPC	**453.77**	525.91	614.87	463.95	513.76	496.32	376.9	831.49	855.5	391.9
	RoA	549.87	497.76	584.14	408.47	505.47	**480.99**	378.38	758.68	744.33	380.39
	AoR	544.33	**490.16**	582.24	**394.25**	**503.01**	487.63	384.07	**735.02**	**725.04**	392.89

Table S.14: Metrics for the simulations varying the number of subpopulations in the W-U scenario.

Metric	Method	Number of Subpopulations				
		2	4	6	8	10
Mean	PiMLE	**99.78**	102.43	102.08	96.96	**103.67**
	MLE	91.61	100.44	**100.48**	96.5	104.22
	MoS	**99.78**	102.43	102.08	96.96	**103.67**
	KLN	91.61	100.44	**100.48**	96.5	104.22
	GNSUM	92.35	100.21	100.5	97.02	103.81
	Overd.	300.19	338.04	419.15	676.39	629.11
	TPC	96.48	104.12	104.65	**100.39**	108.36
	RoA	93.15	100.17	100.52	96.93	104.22
	AoR	93.08	**100.15**	100.64	96.69	103.67
SD	PiMLE	22.85	25.05	26.62	16.72	**26.75**
	MLE	20.51	23.13	26.1	16.17	27.04
	MoS	22.85	25.05	26.62	16.72	**26.75**
	KLN	20.51	23.13	26.1	16.17	27.04
	GNSUM	23	23.03	26.35	16.61	26.76
	Overd.	219.49	298.98	275.79	265.95	273.1
	TPC	20.55	23.45	26.05	**15.72**	27.77
	RoA	20.35	**22.7**	25.95	15.9	27.04
	AoR	**20.14**	22.88	**25.92**	16.27	26.75
MAE	PiMLE	18.74	20.94	21.44	13.78	**21.89**
	MLE	18.86	19.34	21.06	13.63	22.31
	MoS	18.74	20.94	21.44	13.78	**21.89**
	KLN	18.86	19.34	21.06	13.63	22.31
	GNSUM	19.57	19.56	21.23	13.61	22.3
	Overd.	201.38	240.9	319.15	576.39	529.11
	TPC	**16.8**	19.55	20.93	**12.07**	23.73
	RoA	18.06	**18.99**	20.96	13.33	22.31
	AoR	17.85	19.25	**20.91**	13.74	21.89
MSE	PiMLE	495.92	602.06	677.74	274.78	**693.03**
	MLE	469.81	508.22	647.35	260.59	712.5
	MoS	495.92	602.06	677.74	274.78	**693.03**
	KLN	469.81	508.22	647.35	260.59	712.5
	GNSUM	560.89	503.86	659.91	271.05	694.85
	Overd.	85842.11	141584.23	174109.27	399421.74	350814.42
	TPC	**413.61**	539.22	666.47	**234.84**	802.49
	RoA	440.44	**489.6**	639.82	249.45	712.5
	AoR	433.25	497.22	**638.77**	262.59	693.03

Table S.15: Metrics for the simulations varying the number of subpopulations in the W-U scenario with disjoint subpopulations.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	-	-	-	-	-	125.84	-	-	110.37	**101.15**
	MLE	105.48	106.08	**101.3**	103.75	93.32	95.17	**101.1**	100.55	98.34	87
	MoS	-	-	-	-	-	125.84	-	-	110.37	**101.15**
	KLN	105.48	106.08	**101.3**	103.75	93.32	95.17	**101.1**	100.55	98.34	87
	GNSUM	90.26	**105.8**	103.3	107.08	91.2	106.43	95.26	**99.73**	103.42	90.03
	Overd.	165.18	122.07	131.49	122.47	185.06	112.24	123.78	121.96	90.92	132.48
	TPC	113.38	110.91	106.28	108.39	**96.6**	**99.49**	104.93	104.73	102.76	90.75
	RoA	99.58	107.15	101.89	**101.73**	93.29	97.21	101.75	100.65	**99.07**	88.14
	AoR	**99.74**	106.95	102.08	101.84	93.37	97.36	101.19	100.71	98.84	88.03
SD	PiMLE	-	-	-	-	-	37.8	-	-	24.06	22.68
	MLE	24.11	21.43	30.96	17.44	19.67	17.39	14.5	22.06	21.7	18.47
	MoS	-	-	-	-	-	37.8	-	-	24.06	22.68
	KLN	24.11	21.43	30.96	17.44	19.67	**17.39**	14.5	22.06	21.7	**18.47**
	GNSUM	23.33	30.33	30.51	18.97	**18.54**	18.61	20.34	25.72	**20.75**	21.42
	Overd.	139.85	51.47	58.84	70.94	161.67	54.25	54.93	83.03	36.8	79.89
	TPC	25.07	22.66	31.27	18.2	18.7	17.86	**14.39**	**22.05**	22.27	18.76
	RoA	**20.59**	20.41	**29.72**	16.09	20.17	18.38	16.5	23.41	21.09	19.21
	AoR	21.11	**20.18**	30.05	**15.85**	20.6	18.36	16.18	23.26	21.61	19.38
MAE	PiMLE	-	-	-	-	-	33.8	-	-	20.08	17.98
	MLE	19.97	**16.45**	25.56	13.85	17.52	14.61	**11.42**	**17.35**	17.06	17.99
	MoS	-	-	-	-	-	33.8	-	-	20.08	17.98
	KLN	19.97	**16.45**	25.56	13.85	17.52	14.61	**11.42**	**17.35**	17.06	17.99
	GNSUM	21.83	22.58	25.35	16.61	**15.48**	16.25	16.82	20.8	16.47	20.36
	Overd.	85.22	45.02	43.36	45.22	90.83	36.86	37.36	43.63	29.44	56.94
	TPC	22.63	19.66	25.41	15.47	15.69	**13.47**	11.76	17.53	17.61	**16.64**
	RoA	**16.59**	17.97	**24.03**	12.96	16.98	14.61	12.69	19.82	**16.19**	18.84
	AoR	17.07	17.52	24.23	**12.79**	17.12	14.79	12.38	19.43	16.62	18.94
MSE	PiMLE	-	-	-	-	-	2025.12	-	-	657.21	490
	MLE	582.4	473.09	912.33	303.09	412.03	310.69	**200.89**	**462.64**	450.25	492.99
	MoS	-	-	-	-	-	2025.12	-	-	657.21	490
	KLN	582.4	473.09	912.33	303.09	412.03	310.69	**200.89**	**462.64**	450.25	492.99
	GNSUM	611.88	907.37	895.24	392.07	403.91	370.37	415.54	628.3	**420.9**	535.09
	Overd.	22826.77	3004.1	4281.16	5285.25	32065.79	2945.26	3432.13	7031.41	1369.32	7117.77
	TPC	776.13	606.72	968.26	385.05	**343.61**	**303.24**	221.01	484.14	478.91	**420**
	RoA	**403.1**	446.78	**842.51**	248.78	431.59	328.77	261.6	521.21	423.25	491.44
	AoR	423.24	**435.38**	862	**241.91**	447.02	327.28	250.1	514.33	445.13	500.17

Table S.16: Metrics for the simulations varying the number of subpopulations in the W-U scenario with small subpopulations.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	-	-	-	-	-	-	-	118.82	116.23	**102.15**
	MLE	106.19	107.79	102.15	**101.19**	100.85	101.77	108.9	100.42	**101.11**	93.55
	MoS	-	-	-	-	-	-	-	118.82	116.23	**102.15**
	KLN	106.19	107.79	102.15	**101.19**	100.85	101.77	108.9	100.42	**101.11**	93.55
	GNSUM	**100.7**	106.56	**98.8**	103.39	103.42	105.28	**101.66**	100.36	108.17	92.94
	Overd.	130.94	109.99	108.57	124.72	111.43	125.48	159.98	117.76	123.54	126.71
	TPC	111.47	113.74	107.78	105.87	104.97	106.59	113.28	105.16	105.68	97.5
	RoA	102.74	**105.02**	101.47	102.91	101.16	100.42	108.03	100.28	101.89	94.64
	AoR	102.98	105.23	101.37	103.23	**100.66**	**100.27**	107.87	**99.82**	102.08	94.36
SD	PiMLE	-	-	-	-	-	-	-	27.96	21.54	21.56
	MLE	27.2	23.35	19.52	21.65	**22.2**	22.57	26.58	22.95	15.76	19.81
	MoS	-	-	-	-	-	-	-	27.96	21.54	21.56
	KLN	27.2	23.35	19.52	21.65	**22.2**	22.57	26.58	22.95	15.76	19.81
	GNSUM	32.44	34.97	20.42	23.12	23.58	23	24.98	24.67	17.07	21.86
	Overd.	84	35.59	42.81	47.04	45.44	47.77	130.08	44.44	111.05	77.99
	TPC	28.32	24.25	20.84	21.93	22.93	22.73	26.75	23.36	**15.22**	**19.67**
	RoA	**20.02**	19.85	**19.27**	**20.64**	24.09	22.03	**24.81**	20.42	15.68	20.59
	AoR	20.44	**19.76**	19.5	20.65	24.11	**21.76**	25.01	**20.38**	16.5	20.48
MAE	PiMLE	-	-	-	-	-	-	-	27.78	22.09	16.2
	MLE	22.4	18.57	15.31	**17.43**	**16.24**	19.39	22.74	18.27	14.16	15.6
	MoS	-	-	-	-	-	-	-	27.78	22.09	16.2
	KLN	22.4	18.57	15.31	**17.43**	**16.24**	19.39	22.74	18.27	14.16	15.6
	GNSUM	26.54	26.01	**14.32**	18.79	19.44	**17.51**	**20.83**	19.69	15.57	17.95
	Overd.	55.98	28.57	26.64	33.25	35.62	37.35	77.97	38.61	53.67	52.68
	TPC	24.22	20.51	17.38	18.49	17.27	19.6	23.76	19.5	14.81	**15.24**
	RoA	**16.16**	**15.43**	15.24	18.04	17.77	18.75	21.01	16.28	**13.86**	15.97
	AoR	16.62	15.46	15.36	17.92	17.55	18.47	20.92	**16.21**	14.62	15.9
MSE	PiMLE	-	-	-	-	-	-	-	1096.61	704.36	446.03
	MLE	741.01	578.9	366.53	446.53	**468.79**	487.25	750.25	500.69	237.13	414.5
	MoS	-	-	-	-	-	-	-	1096.61	704.36	446.03
	KLN	741.01	578.9	366.53	446.53	**468.79**	487.25	750.25	500.69	237.13	414.5
	GNSUM	999.98	1204.6	397.54	519.48	539.75	530.61	**595.57**	578.28	343.62	503.7
	Overd.	7660.99	1302.75	1814.72	2712.77	2092.4	2816.6	19671.72	2191.23	12269.91	6491.31
	TPC	893.5	747.42	472.92	491.19	524.08	534.21	856.21	545.09	252.38	**373.82**
	RoA	**388.29**	399.65	**354.93**	**413.15**	552.84	461.4	649.02	396.04	**237.06**	431.44
	AoR	405.92	**398.32**	363.24	415.37	552.6	**450.02**	656.08	**394.73**	263.01	430.05

Table S.17: Metrics for the simulations varying the number of subpopulations in the W-U scenario with disjoint and small subpopulations.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	127.87	131.75	128.8	118.91	104.24	98.47	86.18	116.7	104.92	86.04
	MLE	**104.96**	120.43	124.37	117.17	98.9	98.5	84.54	**112.73**	105.01	87.21
	MoS	127.87	131.75	128.8	118.91	104.24	98.47	86.18	116.7	104.92	86.04
	KLN	**104.96**	120.43	124.37	117.17	98.9	98.5	84.54	**112.73**	105.01	87.21
	GNSUM	111.16	125.85	124.33	**117.01**	106.5	97.13	85.09	113.93	**104.5**	85.07
	Overd.	286.15	450.02	524.51	530.21	653.77	722.68	753.97	777.54	809.88	845.64
	TPC	108.86	124.15	129.12	121.65	103.14	102.08	**88.89**	116.72	109.12	**90.96**
	RoA	105.28	**118.33**	**124.07**	117.47	98.85	98.13	84.49	113.15	105.3	87.52
	AoR	105.88	118.78	125.71	117.86	**99.98**	**98.58**	84.41	113.22	105.73	87.94
SD	PiMLE	73.52	81.81	54.21	58.82	57.22	38.84	**37.99**	81.55	**53.11**	35.53
	MLE	59.24	70.65	52.18	58.83	**49.99**	36.33	39.37	**76.04**	53.22	35.71
	MoS	73.52	81.81	54.21	58.82	57.22	38.84	**37.99**	81.55	**53.11**	35.53
	KLN	59.24	70.65	52.18	58.83	**49.99**	36.33	39.37	**76.04**	53.22	35.71
	GNSUM	65.68	73.9	54.16	**56.29**	53.22	36.75	40.7	76.8	54.41	**35.22**
	Overd.	142.51	200.68	234.49	234.02	193.03	181.28	165.76	139.42	105.64	60.66
	TPC	58.87	70.5	52.64	59.14	50.26	**35.95**	39.43	76.47	53.58	35.41
	RoA	**56.72**	**68.46**	**51.71**	58.45	50.19	36.26	39.22	76.37	53.93	35.76
	AoR	57.7	69.89	53.16	58.11	52.22	37.62	38.96	76.25	54.59	36.54
MAE	PiMLE	56.4	68.3	53.24	47.03	40.91	29.64	**32.72**	62.91	39.08	32.76
	MLE	40.8	57.77	49.6	**46.69**	38.28	28.48	33.8	58.95	38.2	32.67
	MoS	56.4	68.3	53.24	47.03	40.91	29.64	**32.72**	62.91	39.08	32.76
	KLN	40.8	57.77	49.6	**46.69**	38.28	28.48	33.8	58.95	38.2	32.67
	GNSUM	44.48	59.79	50.9	47.1	37.7	**28.05**	35.49	59.51	38.34	32.99
	Overd.	186.15	350.02	426.33	430.21	553.77	622.68	653.97	677.54	709.88	745.64
	TPC	**40.44**	57.78	51.97	47.74	**36.94**	28.2	32.74	**58.48**	**37.8**	**31.8**
	RoA	40.65	**56.24**	**49.36**	47.2	38.23	28.41	33.82	59.3	38.27	32.61
	AoR	40.91	56.87	51	47.72	39.15	28.78	33.64	59.57	38.71	33.47
MSE	PiMLE	5912.09	7366.01	3620.49	3644.07	3128.37	1435.44	**1562.11**	6596.88	**2703.76**	1394.23
	MLE	3358.29	5159.76	3180.75	3582.41	**2375.3**	1256.29	1711.29	**5655.17**	2715.46	1374.74
	MoS	5912.09	7366.01	3620.49	3644.07	3128.37	1435.44	**1562.11**	6596.88	**2703.76**	1394.23
	KLN	3358.29	5159.76	3180.75	3582.41	**2375.3**	1256.29	1711.29	**5655.17**	2715.46	1374.74
	GNSUM	4222.72	5855.92	3378.39	**3299.86**	2732.59	1291.28	1795.87	5796.63	2833.07	1400.95
	Overd.	53946.11	160773.21	232439.68	237105.88	342057.09	418947.86	453778.66	477532.97	514531.22	559470.56
	TPC	3370.83	5304.83	3480.41	3791.49	2409.6	**1232.06**	1600.04	5834.45	2810.03	**1272.67**
	RoA	**3084.57**	**4787.77**	**3119.48**	3550.6	2394.59	1252.48	1702.19	5713.86	2791.44	1370.48
	AoR	3197.16	4992.98	3345.57	3527.18	2590.36	1346.73	1684.9	5697.68	2863.78	1414.01

Table S.18: Metrics for the simulations varying the number of subpopulations in the W-S scenario

Metric	Method	Number of Subpopulations				
		2	4	6	8	10
Mean	PiMLE	115.1	104.76	78.48	103.28	104.28
	MLE	97.5	**101.53**	77.62	101.37	**104.12**
	MoS	115.1	104.76	78.48	103.28	104.28
	KLN	97.5	**101.53**	77.62	101.37	**104.12**
	GNSUM	105.1	102.18	77.57	107.04	104.96
	Overd.	272.51	296.45	348.83	619.14	606.8
	TPC	101.57	105.88	**81.55**	105.96	108.09
	RoA	99.02	101.77	77.74	**101.35**	104.12
	AoR	**100.12**	101.93	78.06	101.44	104.28
SD	PiMLE	80.62	42.83	33.21	46.76	**38.1**
	MLE	61.3	42.14	31.93	45.71	38.5
	MoS	80.62	42.83	33.21	46.76	**38.1**
	KLN	61.3	42.14	31.93	45.71	38.5
	GNSUM	67.76	**39.93**	32.41	48.28	38.34
	Overd.	161.24	145.23	155.26	208.73	269.23
	TPC	**61.09**	42.88	**31.83**	**45.43**	38.76
	RoA	63.63	41.21	32.42	45.86	38.5
	AoR	63.85	41.32	32.71	45.8	38.1
MAE	PiMLE	54.77	33.03	31.6	36.07	29.21
	MLE	40.85	32.53	31.32	35.63	**29.11**
	MoS	54.77	33.03	31.6	36.07	29.21
	KLN	40.85	32.53	31.32	35.63	**29.11**
	GNSUM	46.11	32.26	32.26	36.12	29.14
	Overd.	180.84	196.45	251.43	519.14	509
	TPC	**39.91**	33.33	**29.54**	**34.23**	29.63
	RoA	41.58	**32.09**	31.98	35.85	29.11
	AoR	41.67	32.42	31.98	35.37	29.21
MSE	PiMLE	6401.76	1765.73	1510.74	2087.6	**1397.66**
	MLE	3575.92	1689.41	1469.33	**1986.46**	1424.95
	MoS	6401.76	1765.73	1510.74	2087.6	**1397.66**
	KLN	3575.92	1689.41	1469.33	**1986.46**	1424.95
	GNSUM	4387.62	**1519.53**	1501.13	2263.88	1421.24
	Overd.	54458.01	58627.98	84818.53	310894.77	325709.59
	TPC	**3547.35**	1781.56	**1303.01**	1996.2	1492.54
	RoA	3847.01	1616.32	1493.87	2000.06	1424.95
	AoR	3872.49	1625.49	1497.52	1994.53	1397.66

Table S.19: Metrics for the simulations varying the number of subpopulations in the W-S scenario with disjoint subpopulations.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	-	-	-	-	-	154.35	132.45	**99.38**	111.3	131.05
	MLE	92.48	96.38	**102.14**	121.42	101.66	108.52	**105.5**	90.22	93.96	114.95
	MoS	-	-	-	-	-	154.35	132.45	**99.38**	111.3	131.05
	KLN	92.48	96.38	**102.14**	121.42	101.66	108.52	**105.5**	90.22	93.96	114.95
	GNSUM	82.34	89.42	86.89	135.08	107.85	111.95	115.85	84.14	**101.48**	**108.79**
	Overd.	153.23	134.63	134.69	171.19	155.8	125.85	144.9	120.49	149.29	154.97
	TPC	**98.63**	**101.72**	106.98	125.8	105.92	112.57	109.82	94.76	97.54	118.95
	RoA	83.19	96.79	102.25	**121.33**	98.12	105	107.22	91.74	93.78	115.44
	AoR	83.9	97.39	102.94	122.5	**98.82**	**104.8**	106.79	91.54	93.97	117.17
SD	PiMLE	-	-	-	-	-	95.23	76.05	52.71	75.66	90.21
	MLE	45.34	43.35	45.79	75.32	60.45	60.75	**54.21**	45.4	54.86	75.73
	MoS	-	-	-	-	-	95.23	76.05	52.71	75.66	90.21
	KLN	45.34	43.35	45.79	75.32	60.45	60.75	**54.21**	45.4	54.86	75.73
	GNSUM	46.15	**40.58**	**43.04**	94.47	62.85	66.31	59.72	45.14	60.24	71.82
	Overd.	76.81	56.77	52.91	95.41	71.4	59.26	56.49	51	78.81	**43.24**
	TPC	46.45	43.19	46.61	75.4	60.75	60.64	54.64	46.11	**53.8**	75.39
	RoA	**36.88**	42.63	48.3	**74.23**	**57.61**	56.46	56.41	44.91	55.97	79.31
	AoR	37.58	43.07	48.27	76.46	57.69	**56.16**	56.56	**44.46**	55.94	81.43
MAE	PiMLE	-	-	-	-	-	76.14	58.24	41.28	55.54	65.34
	MLE	39.26	34.46	35.03	57.79	44.36	49.79	41.82	37.92	45.06	56.53
	MoS	-	-	-	-	-	76.14	58.24	41.28	55.54	65.34
	KLN	39.26	34.46	35.03	57.79	44.36	49.79	41.82	37.92	45.06	56.53
	GNSUM	38.33	**29.48**	37.94	65.73	42.72	51.74	46.03	40.52	47.71	**55.69**
	Overd.	67.1	52.53	51.18	80.06	67.03	**46.52**	61.9	41.5	61.29	56.1
	TPC	39.01	32.13	**34.57**	58.12	43.18	49.6	**41.44**	37.58	**42.72**	56.1
	RoA	**34.07**	32.82	37.15	**56.22**	**42.05**	47.44	41.83	37.51	45.81	58.36
	AoR	34.42	33.08	36.45	57.2	42.41	47.34	41.85	**37.03**	45.76	59.21
MSE	PiMLE	-	-	-	-	-	11569.97	6547.67	2639.77	5565.25	8694.86
	MLE	2009.32	1798.75	1996.91	5847.97	3473.68	3578.32	**2821.65**	2054.12	2895.57	5671.38
	MoS	-	-	-	-	-	11569.97	6547.67	2639.77	5565.25	8694.86
	KLN	2009.32	1798.75	1996.91	5847.97	3473.68	3578.32	**2821.65**	2054.12	2895.57	5671.38
	GNSUM	2335.12	**1676.58**	**1931.5**	9708.05	3814.54	4320.26	3638.93	2187.58	3450.03	4977.7
	Overd.	8437.82	4260.26	3863.24	13716.66	7957.4	4003.95	5047.25	2891.08	8329.24	**4797.9**
	TPC	2051.8	1774.81	2112.97	6066.73	3541.33	3651.3	2932.43	2047.59	**2755.46**	5758.34
	RoA	**1574.48**	1736.33	2221.07	**5689.65**	**3156.73**	3053.88	3074.75	1984.46	3014.68	6213.84
	AoR	1600.65	1768.87	2221.87	6060.09	3163.06	**3019.24**	3085.19	**1949.65**	3009.65	6593.83

Table S.20: Metrics for the simulations varying the number of subpopulations in the W-S scenario with small subpopulations

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	-	-	-	-	-	-	-	112.48	149.09	-
	MLE	**102.8**	**98.17**	107.53	103	86.94	97.68	110.45	115.61	**123.36**	106.82
	MoS	-	-	-	-	-	-	-	112.48	149.09	-
	KLN	**102.8**	**98.17**	107.53	103	86.94	97.68	110.45	115.61	**123.36**	106.82
	GNSUM	107.35	84.22	86.42	107.11	**96**	103.26	124	**104.69**	138.83	**97.63**
	Overd.	139.66	133.84	152.94	144.11	131.96	149.64	144.72	135.76	131.62	155.76
	TPC	108.63	103.77	112.67	107.27	90.75	**101.67**	114.8	119.75	127.73	111.08
	RoA	106.22	93.03	**107.52**	**101.96**	86.9	95.96	**109.24**	116.29	123.9	103.81
	AoR	106.33	93.49	108.45	102.2	88.01	96.37	110.3	117.06	123.37	104.25
SD	PiMLE	-	-	-	-	-	-	-	49.86	103.21	-
	MLE	40.86	43.69	54.46	45.62	36.28	55.32	42.15	55.25	75.27	44.21
	MoS	-	-	-	-	-	-	-	49.86	103.21	-
	KLN	40.86	43.69	54.46	45.62	36.28	55.32	42.15	55.25	75.27	44.21
	GNSUM	45.09	**37.84**	**44.35**	45	41.72	59.11	46.18	**49.22**	87.73	**39.02**
	Overd.	60.84	76.07	78.94	66.45	37	**53.52**	57.3	53.71	**49.65**	67.86
	TPC	**38.52**	45.34	55.17	46.45	36.16	55.43	41.71	55.74	75.27	44.14
	RoA	40.85	38.8	55.49	**42.51**	**34.96**	54.49	**39.32**	53.62	75.89	42.49
	AoR	42.09	38.94	56.62	43.04	36.28	55.45	39.72	54.56	75.33	42.38
MAE	PiMLE	-	-	-	-	-	-	-	38.23	68.15	-
	MLE	34.83	36.65	41.34	37.36	31.25	47.34	31.2	43.72	49.57	36.08
	MoS	-	-	-	-	-	-	-	38.23	68.15	-
	KLN	34.83	36.65	41.34	37.36	31.25	47.34	31.2	43.72	49.57	36.08
	GNSUM	39.99	35.16	**39.75**	38.38	34.61	48.63	40.04	**37.24**	58.81	**30.65**
	Overd.	48.54	54.24	60.79	60.09	37.7	54.63	50.95	52.94	**47.01**	63.61
	TPC	**33.22**	38	41.43	37.94	**29.51**	46.7	31.43	44.61	50.21	36.63
	RoA	33.8	**32.22**	39.8	**36.19**	31.28	46.63	**30.32**	43.05	50.45	33.74
	AoR	34.21	32.36	40.66	36.54	31.83	**46.62**	30.8	43.75	50.03	33.92
MSE	PiMLE	-	-	-	-	-	-	-	2517.64	12528.68	-
	MLE	1594.25	1816.42	2874.42	1985.8	1420.77	2912.94	1796.72	3144.1	5927.29	1903.62
	MoS	-	-	-	-	-	-	-	2517.64	12528.68	-
	KLN	1594.25	1816.42	2874.42	1985.8	1420.77	2912.94	1796.72	3144.1	5927.29	1903.62
	GNSUM	1985.89	1609.23	**2052.83**	1973.99	1669.8	3330.19	2602.03	**2323.76**	8820.19	**1452.34**
	Overd.	5089.48	6642.59	8722.82	6140.5	2321.8	5184.61	5118.75	4019	**3342.09**	7484.21
	TPC	**1484.04**	1967.57	3052.24	2102.55	**1327.36**	2921.57	1871.7	3341.74	6151.49	1973.79
	RoA	1623.76	**1478.84**	2981.37	**1720.82**	1332.34	**2837.01**	**1554.07**	2996.82	6042.71	1729.92
	AoR	1722.89	1482.69	3117.44	1764.32	1394.34	2933.6	1604.49	3118.54	5937.56	1724.25

Table S.21: Metrics for the simulations varying the number of disjoint subpopulation in the W-S scenario with disjoint and small subpopulations.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	-	**100.68**	89.18	101.57	**100.72**	103.8	97.39	89.59	98.84	**99.92**
	MLE	102.09	96.33	87.36	97.67	93.78	101.22	96.17	88.11	95.74	99.18
	MoS	-	**100.68**	89.18	101.57	**100.72**	103.8	97.39	89.59	98.84	**99.92**
	KLN	102.09	96.33	87.36	97.67	93.78	101.22	96.17	88.11	95.74	99.18
	GNSUM	109.85	106.56	**94.57**	105.8	101.08	102.11	105.04	**103.65**	115.96	97.1
	Overd.	301.29	428.82	516.11	591.77	643.78	685.45	822.84	799.95	794.64	830.13
	TPC	106.82	101.05	91.58	101.71	97.49	105.64	**99.94**	92.04	**99.39**	103.68
	RoA	**98.88**	94.36	88.62	98.64	92.84	**101.14**	97.25	89.04	94.85	99.91
	AoR	101.3	92.7	87.71	**100.64**	96.93	101.62	95.97	88.36	98.1	99.53
SD	PiMLE	-	24.32	26.01	28.4	22.58	23.81	23.39	20.34	24.08	19.39
	MLE	**16.45**	18.74	**22.19**	23.61	16.61	19.19	21.49	18.46	15.62	19.16
	MoS	-	24.32	26.01	28.4	22.58	23.81	23.39	20.34	24.08	19.39
	KLN	**16.45**	18.74	**22.19**	23.61	16.61	19.19	21.49	18.46	15.62	19.16
	GNSUM	34.61	25.33	34.76	39.3	34.24	26.7	30.19	45.78	41.88	24.85
	Overd.	269.91	287.55	328.88	319.02	316.34	289.58	177.85	230.66	213.98	121.15
	TPC	16.92	18.82	22.52	24.22	**15.95**	19.4	**21.23**	**18.37**	15.48	19.48
	RoA	17.29	**17.64**	22.36	**23.23**	16.47	**18.69**	22.11	18.65	**15.03**	**18.79**
	AoR	18.5	21.9	25.11	26.82	20.53	21.65	25.22	19.56	23.56	18.84
MAE	PiMLE	-	17.17	23.27	22.53	17.24	16.66	18.23	18.24	18.99	**14.49**
	MLE	**13.14**	**14.51**	21.11	19.1	14.53	14.16	17.95	19.01	13.14	15.3
	MoS	-	17.17	23.27	22.53	17.24	16.66	18.23	18.24	18.99	**14.49**
	KLN	**13.14**	**14.51**	21.11	19.1	14.53	14.16	17.95	19.01	13.14	15.3
	GNSUM	24.34	19.17	28.88	30.76	25.25	21.47	23.45	33.33	32.25	19.6
	Overd.	201.29	331.45	416.11	491.77	543.78	585.45	722.84	699.95	694.64	730.13
	TPC	14.49	14.59	**20.22**	19.19	**12.81**	14.23	**17.56**	**16.51**	**12.08**	15.41
	RoA	13.44	14.66	20.26	**18.64**	14.62	**13.99**	18.23	18.61	12.82	15.2
	AoR	13.23	18.06	22.11	21.87	15.58	15.81	19.71	17.48	18.63	14.74
MSE	PiMLE	-	562.22	759.91	768.76	484.75	552.77	526.39	501.43	552.4	357.14
	MLE	**261.48**	347.13	627.75	534.89	300.86	351.22	453.48	465.06	249.95	349.45
	MoS	-	562.22	759.91	768.76	484.75	552.77	526.39	501.43	552.4	357.14
	KLN	**261.48**	347.13	627.75	534.89	300.86	351.22	453.48	465.06	249.95	349.45
	GNSUM	1235.12	652.42	1177.61	1500.87	1114.76	681.72	891.1	2004.67	1920.99	594.81
	Overd.	109726.2	186676.67	275897.63	338524.54	390766.7	422411.25	552551.15	540478.68	526025.84	547026.24
	TPC	318.59	337.5	**552.83**	560.2	**248.08**	389.47	**428.36**	**383.93**	**228.14**	373.94
	RoA	285.16	**327.43**	604.33	**514.32**	309.02	**333.18**	472.09	450.49	241.21	**335.55**
	AoR	326.83	508.88	750.34	683.77	409.77	447.9	620.43	498.94	530.99	337.4

Table S.22: Metrics for the simulations varying the number of subpopulations in the P-U scenario.

Metric	Method	Number of Subpopulations				
		2	4	6	8	10
Mean	PiMLE	-	108.87	101.74	85.16	90.75
	MLE	93.2	102.9	98.96	86.35	87.79
	MoS	-	108.87	101.74	85.16	90.75
	KLN	93.2	102.9	98.96	86.35	87.79
	GNSUM	**101.55**	113.61	95.61	**95.81**	87.37
	Overd.	269.9	292.3	382.4	598.85	622.24
	TPC	97.61	106.63	103.53	89.92	**91.71**
	RoA	89.23	**102.17**	98.77	87	87.79
	AoR	89.69	103.31	**100.42**	84.69	90.75
SD	PiMLE	-	27.7	20.73	23.31	29.06
	MLE	17.97	21.83	19.83	**14.99**	22.99
	MoS	-	27.7	20.73	23.31	29.06
	KLN	17.97	21.83	19.83	**14.99**	22.99
	GNSUM	26.96	38.95	22.86	29.35	28.17
	Overd.	185.09	240.04	269.25	277.92	305.96
	TPC	18.47	21.7	19.86	15.12	**22.72**
	RoA	**15.55**	**21.62**	**19.3**	15.38	22.99
	AoR	18.03	26.68	20.86	23.02	29.06
MAE	PiMLE	-	23.49	16.01	20.33	27.35
	MLE	14.82	**16.83**	16.29	14.88	23.06
	MoS	-	23.49	16.01	20.33	27.35
	KLN	14.82	**16.83**	16.29	14.88	23.06
	GNSUM	23.32	27.81	18.87	22.04	25.62
	Overd.	170.11	192.38	282.99	498.85	522.24
	TPC	**13.77**	17.21	15.93	**13.05**	**21.3**
	RoA	14.59	17.06	**15.59**	14.64	23.06
	AoR	15.7	21.79	16.78	20.43	27.35
MSE	PiMLE	-	807.41	411.2	736.53	887.71
	MLE	352.95	461.06	374.53	399.83	651.26
	MoS	-	807.41	411.2	736.53	887.71
	KLN	352.95	461.06	374.53	399.83	651.26
	GNSUM	692.85	1626.33	515.95	835.99	913.51
	Overd.	61412.86	91716.33	148623.92	322221.83	361664.69
	TPC	**329.91**	491.17	387.24	**318.64**	**559.23**
	RoA	345.74	**448.64**	**355.23**	393.75	651.26
	AoR	415.36	687.41	413.67	738.06	887.71

Table S.23: Metrics for the simulations varying the number of subpopulations in the P-U scenario with disjoint subpopulations.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	116.92	91.24	88.36	91.77	87.11	102.32	**100.3**	108.99	97.74	94.21
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	116.92	91.24	88.36	91.77	87.11	102.32	**100.3**	108.99	97.74	94.21
	GNSUM	120.2	124.21	111.07	89.48	89.96	111.74	109.65	120.08	112.54	**99.97**
	Overd.	137.23	137.03	130.87	131.78	139.13	124.89	142.02	**98.06**	141.17	151.28
	TPC	126.39	**96.18**	**92.83**	96.43	90.9	106.78	104.92	113.2	101.89	98.65
	RoA	**92.07**	93.34	91.67	**97.13**	92.07	**99.81**	100.43	102.44	**99.66**	96.07
	AoR	90.38	94.23	90.09	96.31	**92.78**	97.9	103.8	107.36	96.05	94.33
SD	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	33.88	19.92	**17.11**	**18.34**	**20.56**	22.79	24.78	27.74	19.09	22.29
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	33.88	19.92	**17.11**	**18.34**	20.56	22.79	24.78	27.74	19.09	22.29
	GNSUM	54.86	76.35	38.24	25.35	25.74	37.2	39.04	28.76	30.03	31.63
	Overd.	78.32	109.75	62.17	84.18	71.02	77.73	98.95	50.69	74.56	99.45
	TPC	34.67	20.25	17.98	18.47	21.38	23.98	24.91	27.77	18.67	21.89
	RoA	**17.76**	**18.85**	19.09	19.07	21.66	**19.12**	**22.72**	**26.15**	**18.3**	22.51
	AoR	18.3	21.61	18.67	22.29	24.93	24.72	27.09	31.88	19.78	**21.41**
MAE	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	26.36	18.42	17	15.54	20.36	15.78	18.63	22.76	16.15	18.72
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	26.36	18.42	17	15.54	20.36	15.78	18.63	22.76	16.15	18.72
	GNSUM	41.83	46.5	30.75	23.93	23.86	24.21	33.51	27.6	25.04	23.93
	Overd.	49.27	59.82	48.54	52.29	50.02	42.3	67.84	38.85	56.87	69.7
	TPC	31.66	16.13	**15.47**	**13.68**	**18.97**	15.91	18.84	23.45	16.24	**17.67**
	RoA	**15.16**	**15.97**	16.07	14.59	19.38	**14.42**	**17.96**	**20.82**	15.53	18.49
	AoR	16.89	18.24	15.94	16.85	20.86	19.31	23.24	24.43	**15.29**	18.67
MSE	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	1376.63	453.88	413.43	387.41	567.85	498.76	583.27	812.04	351.46	505.61
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	1376.63	453.88	413.43	387.41	567.85	498.76	583.27	812.04	351.46	505.61
	GNSUM	3267.29	6123.1	1511.56	721.23	730.43	1452.55	1540.91	1188.88	1014	950.35
	Overd.	7212.86	12814.88	4624.65	7741.96	6321.89	6359.54	11067.32	2444.55	6975.36	12025.01
	TPC	1838.22	404.3	**358.69**	**336.75**	517.03	592.29	613.81	906.63	334.8	**457.1**
	RoA	**362.66**	**381.79**	415.6	353.59	**508.44**	**347.28**	**490.62**	**655.52**	**318.11**	496.73
	AoR	410.8	477.1	429.33	485.57	642.39	584.93	711.37	1019.46	387.38	467.74

Table S.24: Metrics for the simulations varying the number of subpopulations in the P-U scenario with small subpopulations.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	**103.81**	**102.41**	94.95	86.02	93.82	101.49	92.17	**99.09**	90.4	96.91
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	**103.81**	**102.41**	94.95	86.02	93.82	101.49	92.17	**99.09**	90.4	96.91
	GNSUM	117.1	123.02	115.59	89.25	94.57	115.09	119.33	105.14	110.17	104.89
	Overd.	152.7	137.54	109.31	**97.92**	144.08	105.76	174.86	146.35	**97.76**	84.55
	TPC	110.44	106.78	**100.18**	90.4	**98.2**	106.17	**96.14**	103.15	95.25	**101.02**
	RoA	88.46	104.61	95.92	93.2	96.75	**100.33**	93.03	94.74	89.96	96.59
	AoR	88.36	108.42	98.17	94.12	94.05	101.12	91.73	96.34	89.8	97.51
SD	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	25.08	23.71	19.48	**18.76**	22.37	18.44	17.42	20.83	18.74	22.63
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	25.08	23.71	19.48	**18.76**	22.37	18.44	17.42	20.83	18.74	22.63
	GNSUM	40.55	47.66	28.32	33.86	34.74	31.68	38.46	27.44	36.7	38.63
	Overd.	128.64	138.72	59.32	22.12	113.92	33.21	120.96	124.11	37.63	35.21
	TPC	25.09	24.44	20.13	19.09	22.57	19.04	17.32	21.28	18.59	22.96
	RoA	**24.13**	**23.41**	**16.76**	20.04	22.13	**17.27**	**17.15**	**19.42**	17.24	**22.35**
	AoR	24.23	25.29	20.26	23.34	**21.4**	18.05	18.49	19.79	**15.45**	27.26
MAE	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	21.92	**17.41**	16.31	19.79	17.77	15.4	15.47	17.96	17.51	18.33
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	21.92	**17.41**	16.31	19.79	17.77	15.4	15.47	17.96	17.51	18.33
	GNSUM	33.2	38.45	25.38	29.69	27.66	24.16	33.73	21.29	27.51	29.17
	Overd.	67.79	68.49	37.3	17.87	62.62	24.02	88.98	65.97	25.49	31.04
	TPC	23.22	18.23	16.28	17.28	18.34	16.92	**13.34**	17.93	16	**17.94**
	RoA	**20.83**	18.98	**14.06**	**16.91**	18.59	**14.01**	13.78	16.51	16.9	17.99
	AoR	21.57	20.41	15.48	20.43	**17.39**	14.98	15.68	**16.17**	**14.08**	20.23
MSE	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	**612.17**	**539.94**	385.96	529.66	513.61	325.23	349.64	413.15	425.76	496.09
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	**612.17**	**539.94**	385.96	529.66	513.61	325.23	349.64	413.15	425.76	496.09
	GNSUM	1854.48	2687.43	1005	1205	1176.01	1181.42	1779.27	741.67	1382.85	1441.62
	Overd.	18497.89	19690.53	3429.96	469.14	14272.64	1080.64	19503.24	16781.26	1349.92	1416.13
	TPC	707.2	613.27	384.96	438.22	487.14	382.61	**299.91**	440.19	350.99	501.82
	RoA	686.4	541.89	**283.35**	**427.53**	475.76	**283.55**	328.17	385.72	383.18	**486.24**
	AoR	693.45	678.62	393.28	552.23	**470.6**	310.9	393.23	**385.51**	**330.92**	712.02

Table S.25: Metrics for the simulations varying the number of subpopulations in the P-U scenario with disjoint and small subpopulations.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	-	202.11	198.1	186.16	179.36	185.46	178.14	188.3	184.68	196.19
	MLE	209.76	190.33	188.94	185.33	179.53	184.32	172.08	179.5	180.96	188.12
	MoS	-	202.11	198.1	186.16	179.36	185.46	178.14	188.3	184.68	196.19
	KLN	209.76	190.33	188.94	185.33	179.53	184.32	172.08	179.5	180.96	188.12
	GNSUM	**114.82**	**131.3**	**106.05**	**116.87**	**125.33**	**114.69**	**109.96**	**93.49**	**120.65**	**122.67**
	Overd.	327.7	328.14	492.67	531.49	582.18	571.99	668.86	809.23	737.82	864.59
	TPC	215.15	194.65	192.21	190.2	183.72	188.82	175.77	182.72	185.38	191.72
	RoA	199.71	188.09	188.7	188.8	179.71	184.93	174.9	180.58	179.62	190.64
	AoR	203.14	189.21	191.89	184.95	174.46	183.29	177.61	184.82	181.6	194.49
SD	PiMLE	-	38.71	31.97	30.44	36.06	31.33	27.1	20.31	26.19	34.46
	MLE	32.49	28.44	28.51	27.67	40.75	**29.61**	**17.77**	20.02	29.96	33.39
	MoS	-	38.71	31.97	30.44	36.06	31.33	27.1	20.31	26.19	34.46
	KLN	32.49	28.44	28.51	27.67	40.75	**29.61**	**17.77**	20.02	29.96	33.39
	GNSUM	35.17	42.41	48.06	49.7	81.11	53.29	55.18	32.01	67.94	42.06
	Overd.	178.29	189.13	296.22	238.5	294.89	303.64	255.97	205.24	202.89	143.73
	TPC	32.32	28.78	**28.01**	28.6	40.35	29.62	18.35	**19.85**	30.29	33.41
	RoA	**31.48**	**25.67**	29.56	27.04	39.93	29.86	18.09	19.89	28.25	33.29
	AoR	39.13	34.35	30.42	**26.6**	**33.45**	31.83	25.72	20.2	**23.67**	**32.45**
MAE	PiMLE	-	102.11	98.1	86.16	79.36	85.46	78.14	88.3	84.68	96.19
	MLE	109.76	90.33	88.94	85.33	79.53	84.32	72.08	79.5	80.96	88.12
	MoS	-	102.11	98.1	86.16	79.36	85.46	78.14	88.3	84.68	96.19
	KLN	109.76	90.33	88.94	85.33	79.53	84.32	72.08	79.5	80.96	88.12
	GNSUM	**25.03**	**40.78**	**33.7**	**40.48**	**46.89**	**38.67**	**37.89**	**26.94**	**48.85**	**35.95**
	Overd.	227.7	228.14	392.67	431.49	482.18	471.99	568.86	709.23	637.82	764.59
	TPC	115.15	94.65	92.21	90.2	83.72	88.82	75.77	82.72	85.38	91.72
	RoA	99.71	88.09	88.7	88.8	79.71	84.93	74.9	80.58	79.62	90.64
	AoR	103.14	89.21	91.89	84.95	74.46	83.29	77.61	84.82	81.6	94.49
MSE	PiMLE	-	11849.27	10595.17	8304.52	7534.15	8236.13	6804.05	8188.87	7822.91	10380.13
	MLE	13049.68	8927.35	8682.89	8009.02	7902.75	7943.21	5495.86	6700.77	7407.58	8823.91
	MoS	-	11849.27	10595.17	8304.52	7534.15	8236.13	6804.05	8188.87	7822.91	10380.13
	KLN	13049.68	8927.35	8682.89	8009.02	7902.75	7943.21	5495.86	6700.77	7407.58	8823.91
	GNSUM	**1394.68**	**2688.01**	**2230.96**	**2631.17**	6890.94	**2914.04**	**2992.04**	**1015.77**	**4811.63**	**2194.32**
	Overd.	82046.82	86031.51	237549.51	240219.24	315110.09	310364.3	385842.53	543020.72	445922.46	604218.56
	TPC	14251.63	9745.82	9248.18	8913.73	8555.71	8722.61	6061.14	7216.71	8162.08	9473.73
	RoA	10883.43	8385.51	8697.7	8579.21	7869.11	8060.25	5921.24	6868.96	7098.07	9268.77
	AoR	12092.25	9078.71	9323.43	7889.37	**6607.07**	7899.5	6651.27	7581.22	7190.27	9928.4

Table S.26: Metrics for the simulations varying the number of subpopulations in the P-S scenario.

Metric	Method	Number of Subpopulations				
		2	4	6	8	10
Mean	PiMLE	-	204.13	198.07	193.73	203.36
	MLE	181.31	187.97	193.24	188.16	198.87
	MoS	-	204.13	198.07	193.73	203.36
	KLN	181.31	187.97	193.24	188.16	198.87
	GNSUM	**115.23**	**91.11**	**144.86**	**122.43**	**117.38**
	Overd.	290.83	360.8	465.53	626.91	531.92
	TPC	185.95	192.46	197.25	192.39	203.32
	RoA	174.4	187.37	192.5	187.26	198.87
	AoR	177.83	192.98	193.3	191.27	203.36
SD	PiMLE	-	37.55	36.04	29.63	34.46
	MLE	30.86	**25.89**	31.98	25.61	30.13
	MoS	-	37.55	36.04	29.63	34.46
	KLN	30.86	**25.89**	31.98	25.61	30.13
	GNSUM	55.75	34.26	57.09	64.6	48.13
	Overd.	164.85	219.57	219.32	319.19	301.03
	TPC	30.96	26.31	31.92	25.71	**29.75**
	RoA	**29.66**	25.92	**31.39**	**25.12**	30.13
	AoR	33.1	31.21	33.29	29.59	34.46
MAE	PiMLE	-	104.13	98.07	93.73	103.36
	MLE	81.31	87.97	93.24	88.16	98.87
	MoS	-	104.13	98.07	93.73	103.36
	KLN	81.31	87.97	93.24	88.16	98.87
	GNSUM	**39.89**	**28.39**	**56.81**	**41.94**	**34.21**
	Overd.	190.83	260.8	365.53	526.91	431.92
	TPC	85.95	92.46	97.25	92.39	103.32
	RoA	74.4	87.37	92.5	87.26	98.87
	AoR	77.83	92.98	93.3	91.27	103.36
MSE	PiMLE	-	12181.96	10852.07	9619.05	11811.4
	MLE	7515.54	8376.06	9664.95	8394.66	10637.36
	MoS	-	12181.96	10852.07	9619.05	11811.4
	KLN	7515.54	8376.06	9664.95	8394.66	10637.36
	GNSUM	**3184.38**	**1193.73**	**5109.11**	**4467.75**	**2502.69**
	Overd.	62232.41	113814.41	179302.87	374415.03	272645.25
	TPC	8298.13	9206.32	10426.09	9164.75	11515.77
	RoA	6371.36	8271.31	9492.22	8213.63	10637.36
	AoR	7097.69	9569.89	9758	9162.03	11811.4

Table S.27: Metrics for the simulations varying the number of subpopulations in the P-S scenario with disjoint subpopulations.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	215.27	179.49	200.97	190.67	190.44	170.02	187.56	184.33	176.05	188.15
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	215.27	179.49	200.97	190.67	190.44	170.02	187.56	184.33	176.05	188.15
	GNSUM	**135.09**	**98.98**	**119.68**	**118.76**	**129.26**	**103.48**	**114.01**	**129**	**91.39**	**107.63**
	Overd.	168.69	140.53	200.77	163.57	168.35	137.08	171	138.22	156.06	141.3
	TPC	223.96	184.8	206.34	196.35	195.49	174.88	192.79	188.14	180.39	192.2
	RoA	180.61	188.6	194.6	187.55	195.68	183.4	187.58	178.26	178.68	196.29
	AoR	183.6	189.29	186.23	187.2	197.77	184.12	189.64	179.23	171.31	194.89
SD	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	38.06	33.91	40.78	38.77	35.89	**30.62**	24.31	26.04	26.65	29.74
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	38.06	33.91	40.78	38.77	35.89	**30.62**	24.31	26.04	26.65	29.74
	GNSUM	61.42	43.15	53.5	51.01	53.25	46.49	45.97	81.36	35.01	**29.62**
	Overd.	69.38	45.68	104.08	87.11	69.73	45.19	54.67	52.18	72.62	60.33
	TPC	38.62	34.82	40.17	39	35.68	31.56	25.54	25.13	26.35	30.06
	RoA	**29.35**	**30.67**	39.01	**33.21**	33.87	31.32	**21.14**	24.83	**24.06**	30.68
	AoR	31.92	34.61	**37.37**	33.63	**29.55**	32.91	25.58	**22.59**	30.62	30.58
MAE	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	115.27	79.49	100.97	90.67	90.44	70.02	87.56	84.33	76.05	88.15
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	115.27	79.49	100.97	90.67	90.44	70.02	87.56	84.33	76.05	88.15
	GNSUM	**51.39**	**31.97**	**43.37**	**40.67**	**47.03**	**39.96**	**37.31**	**46.87**	**25.95**	**23.46**
	Overd.	74.57	52.03	103.93	76.26	75.74	46.19	71.99	50.72	72.38	56.16
	TPC	123.96	84.8	106.34	96.35	95.49	74.88	92.79	88.14	80.39	92.2
	RoA	80.61	88.6	94.6	87.55	95.68	83.4	87.58	78.26	78.68	96.29
	AoR	83.6	89.29	86.23	87.2	97.77	84.12	89.64	79.23	71.31	94.89
MSE	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	14662.46	7411.13	11775.2	9647.8	9402.26	5793.32	8227.39	7756.19	6457.54	8610.16
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	14662.46	7411.13	11775.2	9647.8	9402.26	5793.32	8227.39	7756.19	6457.54	8610.16
	GNSUM	**4814.75**	**1770.16**	**3106.67**	**2823.8**	**3550.61**	**2065.07**	**2203.85**	7128.83	**1238.6**	**891.69**
	Overd.	9290.38	3625.08	20447.16	11248.91	9290.78	3314.87	7880.64	**4047.48**	8153.74	5163.41
	TPC	16782.25	8342.51	12841.46	10728.22	10326.92	6552.5	9230.44	8368.55	7122.67	9359.83
	RoA	7315.82	8743.17	10394.01	8713.15	10244.07	7888.16	8094.9	6710.08	6740.34	10166.6
	AoR	7957.61	9111.63	8761.39	8677.98	10389.23	8104.41	8657.03	6762.14	5976.09	9892.51

Table S.28: Metrics for the simulations varying the number of subpopulations in the P-S scenario with small subpopulations.

Metric	Method	Number of Subpopulations									
		2	4	6	8	10	12	14	16	18	20
Mean	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	232.85	178.21	194.73	186.36	185.33	165.49	189.04	197.63	183.06	172.19
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	232.85	178.21	194.73	186.36	185.33	165.49	189.04	197.63	183.06	172.19
	GNSUM	**139.41**	**124.49**	**106.07**	**116.83**	**114.98**	**103.46**	**131.77**	**121.27**	**103.53**	**97.63**
	Overd.	165.02	145.44	162.66	151.73	175.98	150.47	157.68	152.19	149.43	153.31
	TPC	243.98	185.59	200.42	191.37	189.63	169.8	194.34	202.76	188.26	177.31
	RoA	183.78	185.67	188.89	181.99	189.45	172.86	187.01	192.24	181.58	178.53
	AoR	182.33	184.98	188.32	177.36	185.72	176.13	192.65	193.94	181.88	182.33
SD	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	46.05	**29.48**	26.12	34.19	**27.8**	**19.73**	**15.84**	29.75	27.6	**22.36**
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	46.05	**29.48**	26.12	34.19	**27.8**	**19.73**	**15.84**	29.75	27.6	22.36
	GNSUM	43.38	38.53	44.01	57.36	53.81	54.77	56.66	64.85	46.75	39.51
	Overd.	66.76	38.51	62.35	66.09	98.3	67.89	57.5	54.74	52.48	59.33
	TPC	50.5	31.86	26.99	34.97	27.84	20.22	16.39	29.84	28.19	22.4
	RoA	**26.54**	30.8	**23.63**	30.01	28.64	20.86	18.66	**26.44**	**26.3**	24.95
	AoR	30.94	34.93	24.93	**18.36**	32.61	24.8	23.83	28.63	27.79	29.48
MAE	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	132.85	78.21	94.73	86.36	85.33	65.49	89.04	97.63	83.06	72.19
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	132.85	78.21	94.73	86.36	85.33	65.49	89.04	97.63	83.06	72.19
	GNSUM	**42.22**	**35.19**	**30.01**	**38.91**	**36.61**	**39.84**	**42.07**	**49.53**	**34.51**	**30.7**
	Overd.	73.79	46.76	70.05	60.65	76.62	60.79	65.26	61.22	59.49	65.01
	TPC	143.98	85.59	100.42	91.37	89.63	69.8	94.34	102.76	88.26	77.31
	RoA	83.78	85.67	88.89	81.99	89.45	72.86	87.01	92.24	81.58	78.53
	AoR	82.33	84.98	88.32	77.36	85.72	76.13	92.65	93.94	81.88	82.33
MSE	PiMLE	-	-	-	-	-	-	-	-	-	-
	MLE	19664.85	6941.89	9622.56	8569.28	8015.02	4658.97	8166.49	10371.5	7622.29	5686.82
	MoS	-	-	-	-	-	-	-	-	-	-
	KLN	19664.85	6941.89	9622.56	8569.28	8015.02	4658.97	8166.49	10371.5	7622.29	5686.82
	GNSUM	**3340.54**	**2009.89**	**1877.02**	**3408.81**	**2975.12**	**2861.28**	**4059.18**	**4448.4**	**2088.56**	**1488.7**
	Overd.	8461.67	3473.28	7619.21	6825.59	14951.56	6925.4	6468.2	5570.02	5059.47	6186.19
	TPC	23152.8	8289.54	10777.31	9510.99	8770.25	5260.99	9155.72	11405.71	8543.73	6452.99
	RoA	7687.49	8240.73	8431.3	7578.27	8779.83	5721.22	7900.75	9171.78	7312.48	6758.44
	AoR	7687.67	8380.63	8391.22	6304.76	8358.5	6380.46	9122.95	9604.02	7438.02	7603.33

Table S.29: Metrics for the simulations varying the number of subpopulations in the P-S scenario with disjoint and small subpopulations.

Metric	Method	Subpopulations					
		S_0	S_1	S_2	S_3	S_4	S_5
Mean	PiMLE	**99.75**	103.24	-	107.33	102.25	95.35
	MLE	98.3	101.66	**102.79**	102.78	**100.61**	92.67
	MoS	101.66	103.24	-	139.37	142.33	**98.58**
	KLN	98.05	101.66	102.79	**102.11**	100.74	92.59
	GNSUM	97.35	**99.6**	107.56	102.46	103.15	88.69
	Overd.	245.87	614.21	128.65	159.51	207.6	172.65
	TPC	102.41	106.12	107.69	106.83	104.06	96.44
	RoA	97.8	101.44	103.38	103.03	100.85	91.95
	AoR	97.73	101.68	103.79	103.26	100.93	92.01
SD	PiMLE	22.73	27.09	-	21.86	21.32	22.71
	MLE	21.49	25.66	25.32	21.68	20.63	21.69
	MoS	23.08	27.09	-	32.64	45.21	23.67
	KLN	21.31	25.66	25.32	**19.79**	21.16	22.21
	GNSUM	21.32	25.46	26.89	22.35	**20.04**	**19.39**
	Overd.	162.94	331.43	40.39	111.67	128.06	115.88
	TPC	20.92	25.33	25.88	21.68	21.33	21.95
	RoA	20.83	**25.02**	24.19	21.22	20.37	20.77
	AoR	**20.46**	25.23	**24.13**	21.72	20.04	20.83
MAE	PiMLE	18.4	22.18	-	17.67	17.5	16.8
	MLE	17.49	21.14	21.09	15.59	16.87	17.86
	MoS	18.76	22.18	-	41.46	43.96	16.83
	KLN	17.41	21.14	21.09	**15**	17.23	18.61
	GNSUM	18.61	**20.58**	21.66	16.3	**16.24**	18.31
	Overd.	152.9	515.52	36.38	83.59	112.96	79.58
	TPC	17.46	21.3	22.91	16.36	17.3	**16.13**
	RoA	17.34	20.73	**20.46**	15.63	16.84	17.88
	AoR	**17.04**	20.83	20.65	15.92	16.53	17.71
MSE	PiMLE	490.81	707.87	-	507.87	436.88	511.67
	MLE	441.44	628.19	617.08	454.28	404.59	500.71
	MoS	508.7	707.87	-	2561.48	3733.7	534.31
	KLN	435.17	628.19	617.08	**376.63**	426.07	523.69
	GNSUM	438.72	615.92	744.19	480.66	391.5	485.29
	Overd.	46499.64	368771.68	2370.5	15387.1	27156.18	18034.08
	TPC	421.72	647.06	695.14	493.27	448.83	**470.18**
	RoA	417.11	**596.78**	**567.19**	436.94	395	474.63
	AoR	**403.04**	607.44	567.54	458.69	**382.47**	476

Table S.30: Metrics for the simulations varying the sizes of the subpopulations in the W-U scenario.

Metric	Method	Subpopulations					
		S_0	S_1	S_2	S_3	S_4	S_5
Mean	PiMLE	96.42	103.5	-	104.71	94.69	101.99
	MLE	94.05	101.76	98.68	102.23	92.63	**100.04**
	MoS	**98.81**	103.5	-	143.65	125.56	104.06
	KLN	94.05	101.76	98.68	105.24	92.46	100.61
	GNSUM	92.55	98.68	102.99	**98.5**	93.46	98.2
	Overd.	176.83	548.04	123.12	188.26	140.55	204.17
	TPC	98.36	106.06	103.32	105.94	**96.74**	103.88
	RoA	93.62	101.3	99.21	101.89	92.93	100.39
	AoR	94.05	**101.14**	**99.28**	101.82	92.57	100.18
SD	PiMLE	17.73	**19.06**	-	20.83	19.97	22.04
	MLE	16.46	19.63	21.22	19.45	18.13	21.95
	MoS	19.28	**19.06**	-	43.35	43.27	23.15
	KLN	16.45	19.63	21.22	22.27	20.12	22.48
	GNSUM	16.76	19.35	22.17	19.77	18.47	22.8
	Overd.	93.33	300.35	50.25	147.19	69.24	110.19
	TPC	16.81	20.4	21.34	**19.43**	**17.55**	22.25
	RoA	**16.17**	19.33	19.24	19.88	17.83	21.91
	AoR	16.53	19.66	**18.91**	20.11	17.77	**21.67**
MAE	PiMLE	14.36	15.28	-	17.92	15.9	17.07
	MLE	13.31	15.01	15.09	16.2	15.1	**16.63**
	MoS	14.6	15.28	-	47.22	36.89	17.9
	KLN	**13.2**	15.01	15.09	17.84	16.72	17.1
	GNSUM	14.45	**13.65**	16	**15.58**	14.24	18.88
	Overd.	86.5	448.04	35.46	108.92	56.18	109
	TPC	13.25	17.84	13.95	16.8	**13.81**	16.79
	RoA	13.36	14.61	13.93	16.58	14.75	16.77
	AoR	13.59	14.74	**13.73**	16.78	14.92	16.79
MSE	PiMLE	311.34	357.28	-	434.47	407.14	465.27
	MLE	292.9	369.05	429.42	**364.48**	366.68	457.9
	MoS	354.46	357.28	-	3691.17	2431.71	525.57
	KLN	292.62	369.05	429.42	498.41	441.27	480.55
	GNSUM	322.2	357.62	475.93	373.74	366.69	497.08
	Overd.	14178.55	286437.64	2933.58	28371.14	6198.3	22386.66
	TPC	**271.28**	432.04	443.76	393.77	**303.2**	485.45
	RoA	288.98	**356.73**	352.14	378.94	351.96	456.02
	AoR	294.95	368.52	**340.35**	387.43	355.23	**446.25**

Table S.31: Metrics for the simulations varying the sizes of the subpopulations in the W-U scenario with disjoint subpopulations.

Metric	Method	Subpopulations					
		S_0	S_1	S_2	S_3	S_4	S_5
Mean	PiMLE	115.76	113.28	-	133.59	81.51	113.33
	MLE	103.89	116.97	**104.89**	120	82.15	102.54
	MoS	118.32	113.28	-	186.97	120.3	113.04
	KLN	103.82	116.97	104.89	**119.53**	82.38	102.5
	GNSUM	114.93	**108.57**	110.88	127.58	80.78	106.06
	Overd.	197.74	611.61	150.78	160.16	**104.01**	231.04
	TPC	107.9	121.13	108.85	124.54	85.91	106.19
	RoA	**103.48**	117.3	105.64	119.57	83.02	101.1
	AoR	103.86	118.02	105.15	120.66	83.07	**101.02**
SD	PiMLE	63.59	44.73	-	61.16	51.59	50.13
	MLE	50.96	46.2	**69.96**	52.29	52.08	38.86
	MoS	66.54	44.73	-	83.68	132.97	52.15
	KLN	51.04	46.2	69.96	52	**50.95**	38.4
	GNSUM	58.61	**43.82**	75.75	55.71	53.97	39.17
	Overd.	73.43	253.7	87.9	111	58.39	124.81
	TPC	51.11	46.46	70.35	52.05	52.13	38.89
	RoA	50.91	46.71	70.66	**51.98**	53.73	**38.13**
	AoR	**50.86**	46.68	70.56	52.6	53.63	38.9
MAE	PiMLE	44.59	39.53	-	53.86	43.94	40.76
	MLE	38.04	42.02	60.05	43.88	43.45	31.92
	MoS	47.45	39.53	-	92.7	61.63	41.1
	KLN	38.24	42.02	60.05	**43.08**	42.74	**31.12**
	GNSUM	40.5	**36.79**	64.19	48.74	45.66	31.53
	Overd.	98.18	511.61	66.96	75.31	44.3	135.24
	TPC	**36.79**	43.61	59.56	44.34	**41.12**	31.87
	RoA	37.62	42.53	59.74	43.75	43.78	31.24
	AoR	37.85	43.31	**59.51**	44.46	44.03	32.01
MSE	PiMLE	4089.25	2076.81	-	4682.33	2870.55	2564.46
	MLE	2482.57	2315.47	**4673.37**	2997.33	2895.52	1441.39
	MoS	4542.2	2076.81	-	14214.79	17209.55	2753.12
	KLN	2489.76	2315.47	4673.37	2950.27	**2776.63**	1407.08
	GNSUM	3486.48	**1897.59**	5569.42	3709.25	3136.68	1494.23
	Overd.	14676.47	322896.14	9918.4	15325.21	3254.57	31970.14
	TPC	2544.39	2497.09	4780.21	3175.65	2780.24	1474.82
	RoA	2474.42	2372.35	4775.29	**2949.89**	3030.62	**1382.47**
	AoR	**2472.17**	2394.65	4756.38	3054.63	3019.62	1438.5

Table S.32: Metrics for the simulations varying the sizes of the subpopulations in the W-S scenario.

Metric	Method	Subpopulations					
		S_0	S_1	S_2	S_3	S_4	S_5
Mean	PiMLE	90.33	92.87	-	100.97	92.13	116.7
	MLE	84.38	96.97	104.99	96.31	90.1	106.6
	MoS	**92.84**	92.87	-	139.79	134.94	116.61
	KLN	84.31	96.97	104.99	96.75	91.48	107.68
	GNSUM	90.64	90.69	**98.95**	101.49	87.52	112.66
	Overd.	211.22	507.04	127.36	176.64	**104.55**	238.03
	TPC	88.23	**101.58**	109.73	**100.71**	94.36	110.59
	RoA	84.11	97.01	105.39	96.99	90.73	106.54
	AoR	84.73	98.04	105.57	96.37	91.38	**106.08**
SD	PiMLE	40.93	**38.22**	-	44.68	43.52	70.6
	MLE	33.71	41.37	49.78	41.58	42.56	60.24
	MoS	42.89	**38.22**	-	72.74	81.88	69.82
	KLN	33.11	41.37	49.78	**41.26**	44.63	60.57
	GNSUM	35.31	40.36	**39.93**	43.96	42.45	63.78
	Overd.	80.27	214.13	58.43	116.72	52.36	107.42
	TPC	33.99	41.34	50.93	42.12	**42.38**	60.72
	RoA	**33.04**	42.01	49.12	42.54	43.68	60.3
	AoR	33.73	43.19	48.98	42.45	43.07	**59.46**
MAE	PiMLE	36.39	**29.77**	-	36.43	36.04	57.89
	MLE	31.1	32.74	35.58	35.46	36.18	50.72
	MoS	37.89	**29.77**	-	63.76	61.73	56.55
	KLN	30.31	32.74	35.58	35.72	37.07	50.4
	GNSUM	30.86	33.09	**31.74**	35.2	36.46	52.05
	Overd.	116.51	407.04	48.33	96.07	43.75	139.41
	TPC	**30.16**	32.44	35.97	**34.46**	**35.38**	50.4
	RoA	30.66	33.51	36.53	35.71	36.86	50.19
	AoR	31.07	33.89	36.88	35.82	36.84	**49.74**
MSE	PiMLE	1684.59	**1438.4**	-	1897.66	1860.92	5013.24
	MLE	1323.93	1635.45	2379.17	1656.31	1819.01	3490.69
	MoS	1799.19	**1438.4**	-	6609.51	7589.78	4907.43
	KLN	1287.46	1635.45	2379.17	**1627.81**	1964.57	3544.8
	GNSUM	1272.42	1634.31	**1515.52**	1837.89	1868.01	4024.25
	Overd.	18491.34	209245.6	3991.54	18816.88	2624.95	30013.52
	TPC	**1236.11**	1626.17	2558.93	1685.79	**1737.96**	3614.66
	RoA	1289.88	1685.57	2320.83	1728.45	1898.28	3496.46
	AoR	1314.3	1776.17	2310.08	1725.44	1836.52	**3395.99**

Table S.33: Metrics for the simulations varying the sizes of subpopulations in the W-S scenario with disjoint subpopulations.

Metric	Method	Subpopulations					
		S_0	S_1	S_2	S_3	S_4	S_5
Mean	PiMLE	109.32	102.17	-	95.03	91.39	103.71
	MLE	101.57	95.79	95.28	91.4	91.33	97.28
	MoS	109.92	102.17	-	142.36	158.79	108.79
	KLN	101.88	95.79	95.28	90.01	94.98	97.8
	GNSUM	107.73	109.5	92.91	104.23	**100.04**	95.29
	Overd.	190.81	583.27	130.02	108.82	132.2	180.67
	TPC	105.3	**100.1**	**99.8**	**95.77**	94.76	101.45
	RoA	**98.7**	93.65	97.85	92.4	90.7	96.85
	AoR	103.65	97.22	95.07	90.69	88.63	**98.66**
SD	PiMLE	27.6	22.77	-	30.35	28.05	19.94
	MLE	20.86	20.32	21.83	23.69	20.5	19.11
	MoS	28.05	22.77	-	76.81	78.96	23.27
	KLN	20.84	20.32	21.83	24.69	22.32	20.02
	GNSUM	35.49	36.49	24.45	32.04	32.87	**18.35**
	Overd.	124.24	297.1	72.41	74.03	64.79	109.43
	TPC	20.71	20.3	22.19	24	**19.81**	19.8
	RoA	**19.96**	**19.68**	**18.9**	**23.18**	20.79	18.6
	AoR	23.8	21.3	25.13	26.91	25.97	19.42
MAE	PiMLE	23.56	18.7	-	26.06	24.59	15.99
	MLE	16.63	16.81	18.31	20.06	18.43	15.49
	MoS	24.6	18.7	-	60.34	69.05	19.19
	KLN	16.59	16.81	18.31	22.28	19.22	16.13
	GNSUM	25.75	28.57	20.55	22.18	26.79	**14.44**
	Overd.	100.43	483.27	52.4	53.19	53.5	82.88
	TPC	17.18	**16.41**	17.82	19.33	**16.95**	16.15
	RoA	**15.68**	16.62	**14.96**	**18.99**	18.58	15.46
	AoR	19.55	17.36	19.83	23.83	23.33	15.67
MSE	PiMLE	810.53	497.17	-	899.79	821.39	391.56
	MLE	415.81	409.82	474.85	607.32	474.45	354.3
	MoS	846.09	497.17	-	7399.06	9378.23	591.74
	KLN	416.01	409.82	474.85	678.88	498.67	385.57
	GNSUM	1256.6	1355.41	617.99	993.11	1026.56	342.12
	Overd.	22910.74	317400.21	5881.9	5284.62	5024.95	17883.83
	TPC	435.7	**391.53**	467.71	**565.31**	**400.39**	374.64
	RoA	**380.25**	408.36	**343.89**	568.01	496.97	**338.67**
	AoR	551.41	438.81	624.32	774.59	769.74	360.11

Table S.34: Metrics for the simulations varying the sizes of the subpopulations in the P-U scenario.

Metric	Method	Subpopulations					
		S_0	S_1	S_2	S_3	S_4	S_5
Mean	PiMLE	106.51	111.2	-	93.8	103.48	**99.41**
	MLE	**99.91**	103.35	89.86	89.78	**99.52**	93.03
	MoS	109.23	111.2	-	141.37	191.01	102.9
	KLN	100.74	103.35	89.86	88.05	101.38	93.45
	GNSUM	107.98	110.53	**97.56**	93.72	111.96	102.19
	Overd.	193.06	673.1	113.15	**101.54**	157.02	254.67
	TPC	104.51	107.71	94.01	93.69	104.01	97.06
	RoA	97.11	**100.58**	90.69	91.36	98.82	92.69
	AoR	99.67	103.1	91.74	90.01	98.4	94.04
SD	PiMLE	26.42	27.22	-	23.06	26.92	20.23
	MLE	20.22	25.6	16.73	16.21	22.33	13.83
	MoS	29.32	27.22	-	71.42	86.43	24.28
	KLN	20.48	25.6	16.73	**15.24**	23.37	14.37
	GNSUM	35.66	30.37	33.17	25.57	38.09	31.95
	Overd.	133.56	326.04	51.61	63.95	129.68	157.14
	TPC	20.76	25.6	**16.72**	16.95	**22.09**	14.38
	RoA	**19.84**	24.79	18.19	15.64	22.29	**13.6**
	AoR	23.54	**24.46**	17.57	17.5	25.18	19.03
MAE	PiMLE	20.53	24.64	-	18.41	21.54	14.84
	MLE	16.12	21.89	14.66	15.86	17.42	12.71
	MoS	23.75	24.64	-	54.85	101.42	17.73
	KLN	**16.05**	21.89	14.66	15.15	18.36	13.1
	GNSUM	29.99	23.87	27.12	22.03	32.78	25.39
	Overd.	100.75	573.1	39.82	47.28	83.5	157.36
	TPC	16.25	21.85	**13.56**	14.69	17.95	**12.29**
	RoA	16.21	21.38	17	**14.43**	**16.94**	12.59
	AoR	17.71	**21.23**	16.29	15.57	18.46	15.65
MSE	PiMLE	705.56	829.22	-	543.64	700.3	389.08
	MLE	388.25	633.62	368.72	353.85	473.91	230.32
	MoS	901.94	829.22	-	6556.98	15379.35	568.5
	KLN	398.96	633.62	368.72	363.31	520.53	239.2
	GNSUM	1271.63	987.26	1051.43	660.68	1521.08	974.41
	Overd.	25605.73	429435.13	2703.91	3887.3	19226.78	47378.65
	TPC	429.68	682.13	**301.52**	312.77	479.78	**205.19**
	RoA	**382.21**	584.36	400.85	**307.05**	**473.27**	229.29
	AoR	526.37	**577.92**	361.54	390.64	604.72	379.66

Table S.35: Metrics for the simulations varying the sizes of the subpopulations under the P-U scenario with disjoint subpopulations.

Metric	Method	Subpopulations					
		S_0	S_1	S_2	S_3	S_4	S_5
Mean	PiMLE	198.87	192.72	-	193.08	194.74	194.05
	MLE	191.94	184.52	177.16	177.37	183.82	187.66
	MoS	203.56	192.72	-	296.91	304.55	204.7
	KLN	194.15	184.52	177.16	173.04	173.17	191.42
	GNSUM	**121.73**	**108.58**	**115.02**	**102.49**	**107.52**	**113.26**
	Overd.	181.99	471.44	163.24	189.41	201.91	230.81
	TPC	196.42	188.86	180.97	181.71	187.63	192.48
	RoA	187.17	186.91	181.49	180.2	186.49	187.71
	AoR	188.26	188.64	180.65	185.3	192.16	189.04
SD	PiMLE	47.12	32.72	-	43	20.31	31.54
	MLE	35.27	**26.37**	32.66	33.23	**11.78**	25.29
	MoS	51.81	32.72	-	105.72	88.54	35.12
	KLN	36.04	**26.37**	32.66	**32.81**	14.5	26.24
	GNSUM	44.11	55.44	66.27	46.49	38.9	42.9
	Overd.	46.46	290.81	79.95	76.05	116.04	120.44
	TPC	36.06	26.4	32.25	33.07	12.32	25.58
	RoA	**30.82**	28.03	**31.55**	33.35	13.72	**24.98**
	AoR	41.96	31.63	38.15	36.04	20.34	30.3
MAE	PiMLE	98.87	92.72	-	93.08	94.74	94.05
	MLE	91.94	84.52	77.16	77.37	83.82	87.66
	MoS	103.56	92.72	-	196.91	204.55	104.7
	KLN	94.15	84.52	77.16	73.04	73.17	91.42
	GNSUM	**36.73**	**42.54**	**39.15**	**36.3**	**32.18**	**32.25**
	Overd.	83.96	371.44	69.49	94.1	102.59	130.81
	TPC	96.42	88.86	80.97	81.71	87.63	92.48
	RoA	87.17	86.91	81.49	80.2	86.49	87.71
	AoR	88.26	88.64	80.65	85.3	92.16	89.04
MSE	PiMLE	11884.35	9613.67	-	10420.88	9367.17	9790.26
	MLE	9634.81	7804.44	6966.66	7035.64	7158.1	8292.62
	MoS	13274.45	9613.67	-	49391.26	49288.09	12134.26
	KLN	10097.35	7804.44	6966.66	6356.65	5553.4	9012.33
	GNSUM	**2320.79**	**2993.91**	**4397.6**	**2059.43**	**1494.03**	**1924.53**
	Overd.	8773.41	218309.45	10071.31	13488.72	23177.05	30891.84
	TPC	10531.71	8557.97	7544.03	7715.48	7824.15	9173.71
	RoA	8500.24	8299.25	7586.65	7488.05	7658.93	8285.61
	AoR	9461.64	8807.48	7886.54	8509.17	8887.42	8800.86

Table S.36: Metrics for the simulations varying the sizes of the subpopulations under the P-S scenario.

Metric	Method	Subpopulations					
		S_0	S_1	S_2	S_3	S_4	S_5
Mean	PiMLE	204.09	201.22	-	210.75	193.26	199.31
	MLE	198.59	189.66	175.44	188.82	188.8	188.69
	MoS	208.96	201.22	-	298.2	273.42	213.23
	KLN	200.36	189.66	175.44	179.09	178.09	192.68
	GNSUM	**112.92**	**124.96**	**131.72**	**113.71**	**117.87**	**112.42**
	Overd.	203.67	407.38	141.63	198.77	178.61	225.55
	TPC	202.91	193.29	180.63	193	192.8	193.84
	RoA	192.24	191.62	180.63	193.53	191.51	188.45
	AoR	190.21	197.76	186.06	199.18	192.14	192.8
SD	PiMLE	45.95	27.4	-	42.68	36.76	38.22
	MLE	38.84	24.59	38.84	27.71	25.91	35.84
	MoS	49.61	27.4	-	117.98	81.48	41.21
	KLN	40.17	24.59	38.84	30.85	28.15	37.16
	GNSUM	49.77	46.73	62.33	41.53	59.52	53.71
	Overd.	76.47	178.58	58.82	181.6	88.33	184.55
	TPC	39.25	**24.42**	40.33	**26.82**	26.42	36.23
	RoA	**35.38**	24.64	**28.85**	28.61	**25.21**	**35.65**
	AoR	42.84	27.66	30.07	37.86	33.94	37.97
MAE	PiMLE	104.09	101.22	-	110.75	93.26	99.31
	MLE	98.59	89.66	75.44	88.82	88.8	88.69
	MoS	108.96	101.22	-	198.2	173.42	113.23
	KLN	100.36	89.66	75.44	79.09	78.09	92.68
	GNSUM	**36.41**	**40.28**	**47**	**34.58**	**39.15**	**39.8**
	Overd.	107.5	307.38	60.43	113.92	86.13	141.46
	TPC	102.91	93.29	80.63	93	92.8	93.84
	RoA	92.24	91.62	80.63	93.53	91.51	88.45
	AoR	90.21	97.76	86.06	99.18	92.14	92.8
MSE	PiMLE	12839.85	10957.73	-	13996.52	9981.38	11250.07
	MLE	11153.3	8613.34	7123.65	8618.93	8523.19	9085.96
	MoS	14210.77	10957.73	-	52506.71	36380.32	14433.44
	KLN	11605.57	8613.34	7123.65	7159.56	6850.95	9901.77
	GNSUM	**2520.1**	**2697.46**	**4697.28**	**1826.8**	**3685.18**	**2894.57**
	Overd.	16302.9	124775.74	5019.51	41084.65	13590.52	48118.33
	TPC	12053.99	9269.47	8045.69	9332.89	9275.05	10052.01
	RoA	9696.95	8971.04	7292.8	9526.26	8977.83	9031.71
	AoR	9880.75	10283.33	8265.39	11197.46	9584.33	9982

Table S.37: Metrics for the simulations varying the sizes of the subpopulations in the P-S scenario.

Metric	Method	Sample size									
		25	50	100	200	300	400	500	1000	2000	3000
Mean	PiMLE	106.52	110.04	105.37	102.82	102.88	102.14	101.87	101.72	102	101.6
	MLE	103.1	108.21	104.25	102.03	101.66	101.03	100.76	100.56	100.75	100.51
	MoS	110.3	111.08	106.78	104.83	104.74	103.82	104.04	103.65	103.9	103.44
	KLN	103.66	108.25	103.93	102.05	101.71	100.97	100.89	100.62	100.78	**100.44**
	GNSUM	97.24	**103.17**	**100.17**	**98.97**	98.55	**99.7**	**99.83**	98.45	97.93	96.61
	Overd.	203.48	230.55	127.8	106.47	102.19	105.74	101.03	100.82	102.17	101.45
	TPC	111.6	112.77	106.03	102.85	102.23	101.43	101.3	100.78	100.87	100.63
	RoA	**102.18**	108.21	104.08	101.89	**101.35**	101.1	100.65	**100.53**	**100.66**	100.49
	AoR	102.47	108.29	104.29	101.95	101.64	101.27	100.63	100.64	100.73	100.56
SD	PiMLE	36.69	17.36	15.15	11.49	**8.26**	6.87	5.52	3.52	3.97	2.65
	MLE	34.77	16.82	14.28	10.99	8.41	**6.7**	5.54	3.46	3.78	2.51
	MoS	39.27	17.29	16.24	12.23	8.88	7.21	5.67	3.8	4.03	2.7
	KLN	35.27	16.85	14.66	11.01	8.73	6.78	5.67	3.52	**3.74**	2.56
	GNSUM	**31.67**	16.49	**14.06**	**10.51**	9.38	7.71	7.88	5.51	3.86	5.72
	Overd.	129.57	161.15	38.97	11.51	10.18	8.23	6.19	5.57	3.88	2.89
	TPC	35.42	16.84	14.1	10.93	8.39	6.79	**5.44**	**3.33**	3.78	2.54
	RoA	33.84	**16.46**	14.65	10.6	8.81	6.94	5.68	3.51	3.83	**2.44**
	AoR	33.48	16.51	14.87	10.72	8.63	6.85	5.76	3.58	3.84	2.48
MAE	PiMLE	26.84	13.93	12.65	10.24	7.28	5.63	4.65	2.96	3.18	2.5
	MLE	25.76	13.21	11.38	9.45	**6.84**	**5.3**	**4.42**	2.64	2.76	1.9
	MoS	28.92	14.23	13.39	11.13	8.39	6.44	5.62	4.17	4.25	3.68
	KLN	26.26	13.26	11.79	9.43	7.1	5.31	4.5	2.73	**2.74**	1.89
	GNSUM	25.59	12.7	**10.55**	**8.71**	8.3	6.31	6.32	4.23	3.59	5.47
	Overd.	105.02	133.07	29.01	11.1	7.49	7.83	4.6	4.34	3.3	2.52
	TPC	26.09	14.42	11.71	9.47	7.06	5.42	4.47	2.68	2.78	1.98
	RoA	25.39	12.64	11.41	9.12	7.05	5.46	4.51	**2.64**	2.8	**1.86**
	AoR	**25.11**	**12.46**	11.76	9.31	7.04	5.42	4.51	2.66	2.79	1.92
MSE	PiMLE	1321.3	387.17	246.93	133.24	73.05	49.47	32.49	14.75	18.99	9.22
	MLE	1158.04	336.18	211.72	118.78	**69.87**	**43.76**	**29.7**	11.67	14.11	6.26
	MoS	1570.82	407.02	296.43	165.5	97.35	63.97	46.93	27.04	30.61	18.75
	KLN	1195.06	337.59	219.66	119.41	75.3	44.63	31.32	12.16	**13.87**	6.41
	GNSUM	**960.35**	**268.34**	**187.95**	**106.02**	85.67	56.51	59.06	31.26	18.44	42.51
	Overd.	26656.52	41712.57	2215.4	167.67	103.17	97.25	37.43	30.14	18.97	10.04
	TPC	1326.61	432.71	225.16	121.58	71.78	45.87	29.84	**11.13**	14.35	6.55
	RoA	1092.66	325.01	220.57	110.33	75.5	46.94	31.13	11.96	14.35	**5.87**
	AoR	1070.95	327.71	228.59	113.02	73.52	46.24	31.91	12.62	14.53	6.14

Table S.38: Metrics for the simulations of the sample size under W-U scenario.

Metric	Method	Sample size									
		25	50	100	200	300	400	500	1000	2000	3000
Mean	PiMLE	90.38	87.83	101.47	86.5	96.27	99.38	95.53	96.84	97.82	99.27
	MLE	93.77	89.13	101	87.12	97.77	100.38	96.67	98.21	98.46	100.39
	MoS	88.18	85.61	100.9	85.34	95.3	97.23	93.44	95.12	96.22	97.52
	KLN	93.94	89.84	101.08	86.94	97.67	100.28	96.66	98.26	98.49	100.41
	GNSUM	92.14	87.78	98.04	85.12	96.05	97.69	94.17	97.02	95.79	97.21
	Overd.	185.13	196.3	178.21	124.16	115.32	111.42	104.99	102.11	**100.53**	102.47
	TPC	**102.74**	**92.9**	102.99	**88.29**	**98.36**	101.1	**97.14**	**98.68**	98.56	100.42
	RoA	94.69	88.77	**100.44**	87.23	97.83	**100.22**	96.39	98.01	98.33	**100.23**
	AoR	94.53	88.98	100.48	87.13	97.98	100.85	96.82	98.44	98.81	100.66
SD	PiMLE	57.78	45.13	36.75	25.42	19.9	20.67	22.89	8.93	7.52	4.91
	MLE	63.24	43.49	36.16	26.08	20.15	20.92	23.1	9.98	7.77	5.04
	MoS	**52.56**	**41.12**	35.9	**23.89**	**18.89**	**19.57**	**21.83**	**8.55**	**6.96**	**4.8**
	KLN	64.15	44.64	36.25	26.02	19.92	21.01	23.14	9.98	7.78	5
	GNSUM	59.96	42.35	35.43	24.97	19.69	21.18	22.78	10.37	7.77	6.26
	Overd.	94.69	81.48	92.9	50.85	23.2	24.34	24.96	10.24	7.44	5.15
	TPC	65.08	43.66	36.21	26.09	19.94	20.96	23.21	10.11	7.7	5.07
	RoA	63.44	42.87	35.68	26.08	20.27	21	22.83	9.9	7.73	5.1
	AoR	63.03	43.81	**35.09**	26.15	20.45	21.47	23.04	9.97	7.89	5.09
MAE	PiMLE	45.41	36.57	31.52	23.04	16.03	16.89	18.85	**7.56**	6.05	3.88
	MLE	48.51	35.58	30.92	23.01	16.11	16.8	18.6	8.19	6.06	3.88
	MoS	**43.75**	35.18	30.06	22.77	**15.44**	**15.97**	18.69	7.8	6.39	4.35
	KLN	48.74	35.84	30.85	23.1	15.99	16.7	18.69	8.17	6.05	**3.83**
	GNSUM	47.59	35.77	30.28	23.25	16.15	16.59	19.08	8.59	7.18	5.33
	Overd.	92.7	98.77	82.54	39.85	22.37	21.3	19.68	8.52	**5.48**	4.45
	TPC	46.58	**34.58**	30.9	**22.67**	15.89	16.96	18.55	8.39	5.96	3.91
	RoA	48.67	35.39	30.57	22.94	16.18	16.9	18.45	8.09	6.08	3.97
	AoR	48.19	35.53	**29.94**	22.99	16.09	17.24	**18.41**	8.2	6.02	4.1
MSE	PiMLE	3264.49	2082.67	1285.37	796.11	389.98	406.41	517.8	**85.8**	58.55	**23.45**
	MLE	3837.96	1915.17	1243.26	812.25	390.71	415.84	517.99	97.9	59.66	24.25
	MoS	**2764.46**	**1813.66**	1225.21	**757.48**	**361.05**	**371.37**	**495.86**	93.35	60.21	28.08
	KLN	3946.17	1996.48	1249.3	813.66	382.55	419.34	519.94	97.63	59.84	23.93
	GNSUM	3477.35	1853.1	1196.46	813.68	383.95	431.49	526.77	111.06	75.11	44.99
	Overd.	15763.86	15580.89	14315.22	3040.34	745.89	693.06	616.62	104.1	**52.9**	31.29
	TPC	4030.53	1861.6	1254.28	783.85	380.31	418.59	520.09	98.94	58.45	24.57
	RoA	3851.48	1871.88	1209.88	809.43	395.12	418.91	508.28	97.12	59.54	24.75
	AoR	3804.06	1944.55	**1169.99**	815.34	401.39	438.65	514.37	96.83	60.57	25.05

Table S.39: Metrics for the simulations varying the sample size in the W-S scenario.

Metric	Method	Sample size									
		25	50	100	200	300	400	500	1000	2000	3000
Mean	PiMLE	104.27	95.5	97.24	**100.41**	**99.9**	96.8	**99.98**	99.32	98.61	99.35
	MLE	97.19	91.47	94.11	94.9	94.95	92.9	95.35	96.15	94.87	95.32
	MoS	105.6	95.01	**99.03**	101.78	101.1	98.05	101.53	**100.5**	**100.02**	**100.62**
	KLN	96.1	90.59	93.35	94.37	94.49	92.47	94.86	95.61	94.43	94.85
	GNSUM	119.94	**98.95**	102.76	104.88	102.06	**100.28**	104.15	103.95	102.69	105.26
	Overd.	215.37	167.18	108.27	93.59	90.56	88.49	89.41	91.58	88.55	89.32
	TPC	105.02	95.42	96.11	96.03	95.92	93.34	95.87	96.36	94.98	95.37
	RoA	96.82	91.39	93.92	94.22	94.68	92.58	95.08	95.77	94.47	94.95
	AoR	**100.96**	91.48	93.56	95.69	96.91	93.36	96.93	96.76	95.11	95.94
SD	PiMLE	37.65	27.86	18.79	10.45	10.92	9.27	7.69	3.76	3.02	2.39
	MLE	24.51	25.27	15.21	9.22	9.39	6.98	4.52	**3.12**	2.67	1.92
	MoS	38.63	27.63	20.69	10.42	11.57	9.81	7.83	4.17	2.98	2.46
	KLN	24.13	25.03	15.22	**9.12**	**9.09**	**6.95**	**4.34**	3.17	2.66	1.89
	GNSUM	53.55	29.15	19.94	21.54	20.12	24.5	16.67	19.87	19.36	15.75
	Overd.	88.79	92.57	23.07	10.2	9.17	8.43	4.54	5.1	**2.44**	2.57
	TPC	24.3	24.89	15.25	9.43	9.29	7.01	4.61	3.14	2.61	1.92
	RoA	**23.3**	24.93	**14.9**	9.36	9.54	7.01	4.6	3.15	2.72	**1.88**
	AoR	34.08	**24.61**	15.73	10	10.45	8.16	7.11	3.84	2.77	2.09
MAE	PiMLE	28.95	23.77	15.52	8.23	7.93	**7.49**	6.22	**3.04**	2.62	2.01
	MLE	19.19	19.92	13.22	8.45	8	8.34	5.18	4.4	5.44	4.68
	MoS	29.74	23.12	16.42	8.87	8.58	8.06	6.39	3.31	**2.17**	**1.98**
	KLN	19.45	20.12	13.38	8.64	8.05	8.55	5.58	4.79	5.83	5.15
	GNSUM	39.81	25.62	14.2	17.89	14.17	17.9	12.54	15.86	15.74	13.16
	Overd.	122.14	80.55	18.98	9.78	10.98	12.53	10.59	8.86	11.45	10.68
	TPC	19.09	**18.95**	**12.7**	8.09	**7.6**	8.06	**4.89**	4.23	5.32	4.63
	RoA	**18.23**	19.59	13.03	8.95	8.2	8.67	5.33	4.69	5.8	5.05
	AoR	27	21.51	13.95	**7.85**	7.76	8.09	6.3	4.26	5.08	4.06
MSE	PiMLE	1364.95	757.57	343.05	103.89	113.2	91.9	56.23	**13.9**	10.6	**5.86**
	MLE	578.5	679.43	254.58	106.73	109.32	96.78	41.01	24.08	33.05	25.36
	MoS	1449.25	750.42	407.8	106.28	128.36	95.32	60.64	16.8	**8.46**	6.11
	KLN	568.55	683.95	264.47	110.68	108.73	102.56	44.34	28.77	37.74	29.98
	GNSUM	3121.47	808.16	385.32	464.54	388.69	570.21	281.28	390.6	363.27	263.29
	Overd.	20800.7	12653.04	574.06	139.95	169.02	199.82	131.66	95.51	136.86	120.41
	TPC	586.33	**609.34**	**236.08**	**100.16**	**98.69**	**91.01**	**37.29**	22.61	31.64	24.91
	RoA	**526.03**	664.53	247.93	116.65	114.82	101.71	44.27	27.3	37.63	28.81
	AoR	1104.45	648.03	276.64	113.56	113.3	107.33	57.45	24.54	31.23	20.63

Table S.40: Metrics for the simulations of the sample size under P-U scenario.

Metric	Method	Sample size									
		25	50	100	200	300	400	500	1000	2000	3000
Mean	PiMLE	179.43	188.36	185.56	191.24	189.7	190.15	192.99	187.7	192	192.67
	MLE	184.09	183.83	186.24	187.01	183.92	184.24	185.71	183.57	185.33	187.34
	MoS	179.92	192.22	187.78	194.68	193.56	192.93	195.45	190	194.97	195.48
	KLN	181.92	182.7	184.39	186.35	183.34	183.34	184.63	182.4	184.46	186.41
	GNSUM	**114.95**	**105.81**	**108.16**	**88.51**	**116.21**	**117.4**	**104.84**	**123.58**	**117.15**	**106.21**
	Overd.	227.42	188.15	174.64	178.16	172.98	175.92	174.01	172.89	172.65	175.17
	TPC	191.97	188.67	188.37	188.14	184.51	184.87	186.45	183.66	185.4	187.41
	RoA	184.07	183	184.84	186.97	182.95	183.6	185.03	182.69	184.49	186.59
	AoR	174.75	180.36	177.68	185.46	182.84	182.72	185.65	181.24	185.01	185.54
SD	PiMLE	**46.11**	40.51	18.01	18.66	15.57	11.61	9.43	6.92	5.96	3.74
	MLE	49.72	30.42	27.1	**14.05**	12.26	10.06	7.54	7.54	4.89	3.14
	MoS	46.32	40.83	20.5	19.5	16.29	12.2	9.47	6.86	6	3.81
	KLN	48.88	30.16	27.15	14.4	11.84	10.19	7.65	7.41	4.83	**3.14**
	GNSUM	49.82	40.29	41.29	30.99	39.17	44.58	39.63	55.08	37.54	47.83
	Overd.	164.27	85.13	27.59	18.51	15.29	13.89	11.26	10.63	6.18	3.86
	TPC	49.87	**29.68**	27.62	14.23	12.73	9.9	**7.35**	7.46	4.77	3.31
	RoA	48.63	30.1	25.61	14.47	**11.76**	**9.77**	7.59	7.73	**4.64**	3.16
	AoR	46.29	39.01	**15.57**	16.89	13.03	9.93	9.03	**6.84**	5.34	3.29
MAE	PiMLE	79.99	88.36	85.56	91.24	89.7	90.15	92.99	87.7	92	92.67
	MLE	84.09	83.83	86.24	87.01	83.92	84.24	85.71	83.57	85.33	87.34
	MoS	81.03	92.22	87.78	94.68	93.56	92.93	95.45	90	94.97	95.48
	KLN	81.92	82.7	84.39	86.35	83.34	83.34	84.63	82.4	84.46	86.41
	GNSUM	**30.93**	**30.18**	**31.07**	**28**	**31.37**	**35.71**	**29.52**	**44.56**	**28.81**	**30.58**
	Overd.	132.87	94.34	74.64	78.16	72.98	75.92	74.01	72.89	72.65	75.17
	TPC	91.97	88.67	88.37	88.14	84.51	84.87	86.45	83.66	85.4	87.41
	RoA	84.07	83	84.84	86.97	82.95	83.6	85.03	82.69	84.49	86.59
	AoR	75.15	80.36	77.68	85.46	82.84	82.72	85.65	81.24	85.01	85.54
MSE	PiMLE	8329.65	9367.02	7629.46	8654.48	8276.41	8254.53	8732	7737.05	8497.51	8601.07
	MLE	9419.88	7905.97	8134.63	7758.25	7184.69	7192.93	7400.63	7038.26	7303.86	7637.86
	MoS	8425.2	10088.98	8103.89	9325.5	9005.92	8776.65	9196.55	8143.86	9053.25	9129.65
	KLN	8981.4	7702.94	7821.97	7653.44	7078.17	7044.16	7217.75	6842.41	7154.97	7475.75
	GNSUM	**2581.1**	**1576.19**	**1686.4**	**1044.34**	**1720.59**	**2190.76**	**1515.66**	**3438.25**	**1633.2**	**2211.54**
	Overd.	41872.61	14655.12	6294.55	6434.64	5547.58	5947.32	5597.73	5420.24	5314.36	5664.1
	TPC	10821.37	8698.37	8533.1	7961.89	7295.43	7296.05	7524.08	7051.28	7315	7650.38
	RoA	9314.68	7750.66	7821.48	7763.26	7011.28	7079.23	7284.34	6893.68	7159.68	7507.74
	AoR	7623.21	7902.53	6265.1	7574.88	7024.41	6935.68	7413.61	6643.74	7253.64	7327.13

Table S.41: Metrics for the simulations varying the sample size in the P-S scenario.

5 Case study

We present the performance of the estimators using a real network of the Deezer dataset. Table S.42 shows the values of the mean, standard deviation, relative mean absolute error (RMAE), and mean squared relative error (MSRE) when the subpopulations are simulated uniformly. Table S.43 displays the values for the metrics using a SIR process to generate the unknown subpopulation, while the known subpopulations are simulated uniformly. The mean, standard deviation, RMAE, and MSRE using the real subpopulations are shown in Table S.44.

Metric	Unknown subp.	Methods								
		PiMLE	MLE	MoS	KLN	GNSUM	Overd.	TPC	RoA	AoR
Mean	Asian (503)	-	482.92	-	484.99	504.39	1789.53	513.68	486.24	501.71
	Blues (2969)	-	2888.98	-	2909.69	3019.88	2991.59	2922.8	2908.96	2959.95
	Chillout (875)	-	864.93	-	871.35	891.85	1703.52	893.67	866.26	913.89
SD	Asian	-	133.42	-	134.1	175.88	1157.6	131.59	135.36	219.39
	Blues	-	293.97	-	304.86	766.11	407.2	296.3	299.81	435.91
	Chillout	-	135.47	-	141.99	238.95	757.15	134.12	135.55	249.62
RMAE	Asian	-	0.19	-	0.19	0.24	2.56	0.18	0.19	0.29
	Blues	-	0.07	-	0.07	0.2	0.1	0.07	0.07	0.11
	Chillout	-	0.13	-	0.13	0.2	0.97	0.12	0.13	0.23
MSRE	Asian	-	0.05	-	0.05	0.09	11.09	0.05	0.05	0.16
	Blues	-	0.01	-	0.01	0.06	0.02	0.01	0.01	0.02
	Chillout	-	0.02	-	0.02	0.04	0.94	0.02	0.02	0.05

Table S.42: Metrics of the estimates of several unknown subpopulations, with the subpopulations simulated uniformly over the Deezer network. The numbers in parentheses represent the sizes of the subpopulations.

Metric	Unknown subp.	Methods								
		PiMLE	MLE	MoS	KLN	GNSUM	Overd.	TPC	RoA	AoR
Mean	Asian (539)	-	1465.97	-	1461.68	573.25	2453.03	1495.36	1465	917.78
	Blues (3495)	-	7600.44	-	7607.82	3693.12	7699.17	7636.34	7595.15	5589.19
	Chillout (1099)	-	2354.46	-	2346.53	1129.09	2859.07	2386.83	2333.42	1756.84
SD	Asian	-	373.11	-	370.01	220.08	1020.38	376.43	373.96	265.31
	Blues	-	1038.47	-	1021.66	724.8	1216.2	1031.14	1021.11	796.22
	Chillout	-	441.04	-	433.58	235.25	858.43	440.45	433.91	409.62
RMAE	Asian	-	1.72	-	1.71	0.32	3.55	1.77	1.72	0.72
	Blues	-	1.17	-	1.18	0.16	1.2	1.18	1.17	0.6
	Chillout	-	1.14	-	1.14	0.19	1.6	1.17	1.12	0.6
MSRE	Asian	-	3.43	-	3.4	0.17	16.33	3.62	3.42	0.71
	Blues	-	1.46	-	1.46	0.05	1.55	1.48	1.45	0.4
	Chillout	-	1.3	-	1.29	0.04	2.56	1.37	1.26	0.36

Table S.43: Metrics of the estimates of several unknown subpopulations, with the known subpopulations simulated uniformly and the unknown subpopulation modeled via a SIR process over the Deezer network. The numbers in parentheses represent the sizes of the subpopulations.

Metric	Unknown subp.	Methods								
		PiMLE	MLE	MoS	KLN	GNSUM	Overd.	TPC	RoA	AoR
Mean	Asian (503)	-	516.1	-	530.06	512.65	1991.01	545.29	527.47	499.64
	Blues (2969)	-	2407.57	-	2451.13	2644.8	3081.55	2440.29	2455.69	2960.41
	Chillout (875)	-	781.84	-	798.08	699.18	2286.68	812.95	792.57	946.78
SD	Asian	-	122.19	-	131.4	196.58	1116.22	122.54	121.19	182.23
	Blues	-	306.87	-	286.19	801.9	744.22	307.57	335.37	468.75
	Chillout	-	172.79	-	170	233.49	1469.69	173.74	186.92	285.75
RMAE	Asian	-	0.19	-	0.2	0.28	2.96	0.2	0.19	0.28
	Blues	-	0.19	-	0.17	0.27	0.16	0.18	0.18	0.13
	Chillout	-	0.19	-	0.19	0.28	1.63	0.18	0.2	0.28
MSRE	Asian	-	0.05	-	0.06	0.11	14.24	0.06	0.05	0.12
	Blues	-	0.04	-	0.04	0.1	0.06	0.04	0.04	0.03
	Chillout	-	0.04	-	0.03	0.08	2.65	0.03	0.04	0.08

Table S.44: Metrics of the estimates of several unknown subpopulations, using real subpopulations of the Deezer dataset. The numbers in parentheses represent the sizes of the subpopulations.

6 Comparison between estimators based on known subpopulations and based on direct degree estimation

This section displays the best method for each scenario in the case of the variation of the number of known subpopulations and the sizes of the known subpopulations. We include one table that considers all estimators and another table that excludes the estimators that do not use the known subpopulations, namely RoA and AoR. We observe that the best estimator using the known subpopulations is very close to the estimators which have the real information of the network size.

Simulation goal	Main scenarios	Mean	MAE	MSE
Subpop. number	W-U	MLE-KLN/100.00	MLE-KLN/18.36	MLE-KLN/530.82
	W-S	MLE-KLN/105.38	TPC/42.39	MLE-KLN/3036.94
	P-U	TPC/99.93	TPC/15.71	TPC/382.1
	P-S	GNSUM/115.58	GNSUM/ 37.52	GNSUM/ 2976.36
Subpop. sizes	W-U	MLE-GNSUM/99.80	MLE/18.34	KLN/501.14
	W-S	KLN/105.02	KLN/42.87	KLN/2768.76
	P-U	TPC/99.53	TPC/17.31	TPC/439.21
	P-S	GNSUM/111.43	GNSUM/ 36.52	GNSUM/ 2531.72

Table S.45: Best method for each scenario. For each metric, it shows the average of the values corresponding to each parameter. AoR and RoA are not included in the simulations.

Simulation goal	Main scenarios	Mean	MAE	MSE
Subpop. number	W-U	MLE-KLN/100.00	AoR/18.24	MLE-KLN/523.87
	W-S	RoA/105.26	TPC/42.39	RoA/2976.75
	P-U	TPC/99.93	TPC/15.71	TPC/382.1
	P-S	GNSUM/115.58	GNSUM/ 37.52	GNSUM/ 2976.36
Subpop.sizes	W-U	AoR/99.90	AoR/18.11	AoR/481.28
	W-S	RoA/105.02	KLN/42.87	KLN/2768.76
	P-U	TPC/99.53	RoA/16.72	RoA/422.69
	P-S	GNSUM/111.43	GNSUM/ 36.52	GNSUM/ 2531.72

Table S.46: Best method for each scenario. For each metric, it shows the average of the values corresponding to each parameter. All estimators are included in the simulations.

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