

A Many-Objective Optimization Approach for Weight Gain and Animal Welfare in Rotational Grazing of Cattle

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Abstract

The “multidimensional” nature of the concept of welfare is reflected in the definition proposed by the World Organization for Animal Health (OIE), according to which an animal is in a satisfactory state of welfare when it is healthy, comfortable, and well-fed, can express its innate behavior, and does not suffer pain, fear, or distress. Many of these aspects, in the real context of a cattle farm, are not considered, **and most of the farmers’ decisions are based on their experiences. In this proposal, we establish a many-objective optimization model for rotational grazing allocation based on six objectives that consider cattle weight gain and travel, as well as their welfare.** The model is solved using the NSGA-III algorithm, and its performance is evaluated using a simulation study of 90 days of rotational grazing in which it is compared with the traditional grazing strategy. Average weight gains of up to 36.7 kg per animal are achieved at the end of the three months of simulated grazing using the proposed model. The results indicate that the allocation model generates **an average weight gain that is statistically greater than that generated by the traditional rotation method but also guarantees improved animal welfare, the main contribution of our approach.**

keywords: Many-objective Optimization, Artificial Intelligence, Precision Livestock Farming, Animal Welfare, Rotational Grazing

1 Introduction

The concept of animal welfare includes three elements: the proper functioning of the organism (which implies, among other things, that the animals are healthy and well-fed), the emotional state of the animal (including the absence of negative emotions such as chronic pain and fear), and the possibility of expressing some normal species-specific behaviors [1].

30 According to the so-called principle of the five freedoms, the welfare of an animal is guar-
31 anteed when the following five requirements are met [2]: the animal does not suffer from
32 thirst, hunger, or malnutrition because it has access to drink water and is provided with a
33 diet adequate to its needs, the animal does not suffer physical or thermal stress because it
34 is provided with a suitable environment, including shelter from inclement weather and a
35 comfortable resting area, the animal does not suffer pain, injury or disease thanks to ade-
36 quate prevention and/or rapid diagnosis and treatment, the animal can exhibit most of its
37 normal behavioral patterns because it is provided with the necessary space and adequate
38 facilities, and is housed in the company of other individuals of its species, and the animal
39 does not experience fear or distress because the necessary conditions are guaranteed to
40 avoid mental suffering. The principle of the five freedoms constitutes a very useful practi-
41 cal approach to the study of welfare, and especially, to evaluate these aspects on livestock
42 farms and during the transport and slaughter of farm animals.

43 On the other hand, one way of feeding cattle is using rotational grazing; this type of
44 grazing has been used in livestock farming for many years, and has been recognized as a
45 more efficient and sustainable alternative to continuous grazing [3]. Rotational grazing is
46 a strategy used by livestock farms, dividing their land into smaller plots through the use of
47 electric or wire fencing. Its main objective is to achieve a balance between pasture supply
48 and the nutritional needs of livestock [4]. In situations where the same amount of pasture
49 is available, rotational grazing allows a greater number of cattle to be maintained, resulting
50 in higher productivity [3]. In addition to natural factors, overgrazing is one of the main
51 causes of degradation of rangeland ecosystems [5]. Rotational grazing presents itself as a
52 reasonable option to combat overgrazing, as it helps to increase rangeland productivity and
53 improve ecosystem functionality. Generally, the periods of occupancy, rest, and allotment
54 in rotational grazing are determined based on the subjective experience of livestock farmers
55 [6]. High-quality forage management together with animal welfare are some of the current
56 limitations on cattle farms highlighted in a recent systematic review of the literature [7].

57 1.1 Related Works

58 Depending on the number of objectives, an optimization problem is referred to as single-
59 objective, multi-objective, or many-objective [8]. When a multi-objective problem has a
60 large number of objectives (usually more than 4) it is classified as a many-objective opti-
61 mization problem [9,10]. With respect to the many objectives optimization problem, Raoui
62 *et al.* [11] proposed to address the problems of high-demand and low quality in perishable
63 food distribution through a customer-centric mathematical model that considers deliv-
64 ery times, destination times, and customer priorities. They use a heuristic approach called
65 General Variable Neighborhood Search, which generates multiple solutions and ranks them
66 according to the decision maker's preferences. The results show that this approach gener-
67 ates high-quality solutions and allows different rankings according to the decision maker's
68 profiles. The scientific contributions include the ability of general variable neighbour-
69 hood search to generate high quality and efficient generation of many candidate solutions.
70 However, the study lacks environmental features, such as CO2 emissions reduction in the
71 proposed model.

72 Jafar *et al.* [12] described a common problem in watershed management, where the
73 complexity of water resource systems, the difficulty of high-dimensional modeling, and
74 computational efficiency challenges limit the ability of decision-makers to combine envi-
75 ronmental flow objectives (e.g., water quality) with social flow objectives (e.g., hydropower,
76 or water supply). They developed a watershed management decision support tool called
77 Optimum Social-Environmental Flows with Auto-Adaptive Constraints. This approach
78 integrates nine socio-environmental objectives and 396 decision variables into a watershed

79 management model of the Diyala River basin in Iraq. Their contribution is to use evolu-
80 tionary optimization algorithms, such as the e-DSEA algorithm and the Borg MOEA, to
81 address the complexity of reservoir and catchment management in terms of non-linearity,
82 considering dynamic characteristics. However, their mathematical optimization model does
83 not use characteristics such as lake water inflow, and reservoir water inflow, among others.

84 Chikumbo *et al.* [13] addressed the land use optimization problem for a large farm, con-
85 sidering 14 objectives including economic, environmental, and social aspects. They used
86 a modified non-dominant sorting genetic algorithm II (NSGA-II), and the solution was
87 represented as a hyperspatial Pareto frontier, which was collapsed into a two-dimensional
88 visualization using a hyperradial visualization approach. Their contributions include the
89 development of a transdisciplinary approach that integrates an innovative epigenetics-
90 based multi-objective optimizer, the incorporation of uncertainty in search space data,
91 and decision-making through visualization of the three-dimensional exchange space. The
92 approach allowed decision-makers to intuitively select a compromise solution based on their
93 preferences under uncertainty. Nevertheless, the study does not focus on specific regions
94 of the Pareto frontier in the process of searching for desired solutions.

95 White *et al.* [14] developed a model that optimizes pasture and nutritional management
96 to examine the environmental impact of beef production. White *et al.*'s model integrated
97 modules that calculate (1) environmental impact from cradle to the farm gate, (2) diet
98 cost, (3) pasture growth, and (4) willingness to pay. Their contribution was to use different
99 objectives, including the minimization of the cost of the diet, and the minimization of the
100 environmental impact metrics regarding the baseline value, among others. However, more
101 accurate pasture simulation models should be used to accurately simulate the heterogeneity
102 of the landscape.

103 Raizada *et al.* [15] used multi-objectives to develop alternative land use plans to opti-
104 mize four objective functions: maximizing (1) farm income, (2) employment (3) nutritional
105 security and (4) forage production, and minimizing (1) soil loss (2) watershed level loss, to
106 guarantee a sustainable animal population. The main contribution of this work is the use
107 of modeling methods and paradigms in multi-criteria decision analysis for natural resource
108 management. They also incorporated temporal and spatial environmental data. Addis
109 *et al.* [16] developed a profit optimization model for a silage supplementation scenario.
110 They employed linear programming to identify the optimum carrying capacity of cattle
111 and sheep, the most profitable slaughter ages of cattle, the number of prime lambs (sold to
112 meat processing plants), and the reserve lambs sold (sold to other farmers for finishing).
113 The contribution is the use of optimization to maximize resource allocation efficiency by
114 identifying the optimum number of cattle and sheep that can be managed within the avail-
115 able feed resources, considering strategies such as early finishing of cattle and selling the
116 majority of sheep at their best time. This study lacks research on pasture quality man-
117 agement, the use of breeding cows, and the assessment of uncertainty and risk in model
118 decisions.

119 Zhai *et al.* [17] proposed a drone mission planning algorithm, which combines Genetic
120 Algorithms and Particle Swarm Optimization, treating the planning problem as a Multi-
121 Objective Optimization problem. Through simulations, they demonstrated the feasibility
122 of the approach in achieving efficient mission planning and optimal resource allocation.
123 Their main contribution is to use a multi-agent system where components, such as UAVs,
124 are considered autonomous agents. Validation through simulations, such as the "precise
125 pesticide spraying" task, supports the effectiveness of the approach by demonstrating the
126 ability to generate optimal mission planning strategies, considering aspects such as ex-
127 pected profit, energy consumption, and equipment loss.

128 Li *et al.* [18] developed an integrated modeling framework based on the water-energy-
129 food nexus to maximize agroforestry-livestock system performance under uncertain water

130 supply conditions. Using a multi-objective programming approach and empirical frequency
131 analysis for different water supplies. The model addressed the complex interrelationship
132 between energy and material conversion processes on agricultural, forestry, and grazing
133 lands. Their contributions include a systematic analysis of energy flows and material
134 conversion, consideration of trade-offs between economic benefits, efficiency of multiple
135 energy use, and environmental and ecological benefits. Michalak *et al.* [19] approached the
136 multi-objective optimization of neural models to make decisions on vaccine distribution in
137 a scenario of disease spread between farms, pastures, and other locations. Three neural
138 models were analyzed: multilayer perceptrons, classical recurrent neural networks, and
139 short- and long-term memory networks, whose weights were optimized using the MOEA/D
140 algorithm.

141 Chen *et al.* [20] proposed an optimization model-based evaluation method for config-
142 uring integrated crop-livestock systems to improve agricultural sustainability. The Op-
143 timization Model-based Energy Evaluation method combines an energy analysis with a
144 non-dominated genetic algorithm NSGA-II programming model. Using economic energy
145 efficiency, environmental energy efficiency and energy sustainability indexes, sustainable
146 development is evaluated. The contribution of this work is the definition of theoretical
147 guidance for quantitative resource allocation in integrated farming systems.

148 Castonguay *et al.* [21] et al. developed a multi-objective optimization tool for livestock
149 production, addressing economic and environmental objectives in agriculture and animal
150 husbandry. Using advanced techniques, such as high granularity spatial optimization,
151 the model evaluates trade-offs between reducing greenhouse gas emissions and minimiz-
152 ing production costs in beef production. Finally, Shahin *et al.* [22] used multi-objective
153 optimization algorithms and IoT data mining, to calculate farm-level greenhouse gas emis-
154 sions. They proposed optimized feeding schedules to mitigate emissions. The application
155 is based on a case study on a dairy farm and is positioned as a valuable tool for sustainable
156 emissions management in livestock production.

157 In relation to some recent works that study the relationship between crop/feeding
158 optimization versus animal health, Erinle *et al.* [23] presented a review of the applicability
159 and impact of fruit pomaces in poultry nutrition. They concluded that the utilization
160 of plants and/or their by-products, like fruit pomaces, has important advantages. They
161 have a rich nutritional composition and phytochemical profile, and are ready availability
162 and a pocket-friendly cost. Particularly, fruit pomaces contain protein, dietary fiber, and
163 phenolic compounds, and thus, can be used by the poultry industry as a substitute for
164 antibiotics and some conventional feedstuff. Also, Mallick *et al.* [24] proposed a linear
165 programming technique to minimize the feed cost for small-scale poultry farms. This
166 approach uses locally available feed ingredients to formulate the broiler feed mix. The
167 dietary nutrient requirements for broilers are determined from the prescribed standard
168 specifications by international standard institutions and sixteen feed ingredients were used
169 to formulate the optimal feed mix, minimizing the total cost of the feed mix subject to
170 the essential nutrient constraints. Alqaisi *et al.* [25] proposed a static linear programming
171 approach for the sustainable feed formulation for crop farmers and livestock producers.
172 The diet formulation defines nutritional and economic feed optimization considering the
173 interaction between feed components over time and the volatile global feed prices.

174 The work of Han *et al.* [26] proposed a simulation of the system dynamic of herbivorous
175 animal husbandry in agricultural areas. They studied the development of herbivorous
176 animal husbandry, and the balance of livestock-grassland as a constraint. The system
177 designs the development strategy to optimize the herbivorous animal husbandry and the
178 feed planting industry. They found that without any development strategy, the inertia of
179 the system is subject to factors such as the scale of female livestock and epidemic diseases,
180 among other factors. The paper of Dooyum *et al.* [27] presented the problem of feed

181 formulation in the context of the livestock industry as a hard (NP-hard) problem. The
182 feed formulation is defined by specifying the nutritional requirements as rigid constraints to
183 find a feasible cost-effective formulation. They modified the conventional problem with a
184 tolerance parameter to allow the relaxation of constraints and used the differential evolution
185 technique, a type of evolutionary algorithm, to solve the problem.

186 Gharehchopogh et al. [28] define a population evolution strategy to help the multi-
187 population evolution algorithm improve its global optimization ability and avoid local
188 optimum. They compare this approach with five state-of-the-art variants and seven basic
189 metaheuristic algorithms over 30 benchmark functions. The paper [29] introduces a binary
190 multi-objective dynamic Harris Hawks Optimization (HHO) applied to Botnet Detection
191 in IoT. They improve HHO with a mutation operator to obtain better performance over
192 other machine learning approaches.

193 As can be seen in the review of the literature, the many-objective models that have been
194 proposed have been dedicated to solving problems such as food distribution, watershed
195 management, land use, or pasture and nutritional management, among others. On the
196 other hand, multi-objective optimization models have been proposed to optimize livestock
197 that can be managed within available food resources and maximize the performance of
198 the agroforestry-livestock system under uncertain water supply conditions, among others.
199 That is to say, there are no works that propose many-objective models that allow, in
200 addition to improving the fattening of livestock, their welfare.

201 1.2 Contributions

202 The focus of this work is on the use of beef production variables for optimal grazing deci-
203 sions while maintaining animal welfare, with a focus on autonomous or semi-autonomous
204 beef production that can be included in autonomous cycles of data analytics tasks (ACO-
205 DAT) [30, 31]. The ACODAT is a great help in corrective decision-making because it
206 generates knowledge to determine decisions that favor the performance of beef produc-
207 tion [32, 33]. Specifically, the objective of this work is to define a dynamic optimization
208 model for the daily allocation of lots of animals to pastures, which can be included in
209 an autonomous system for managing the production process of cattle fattening. Thus,
210 this paper presents a rotational-grazing assignment model that seeks to maximize animal
211 weight gain based on the best quality forage and animal welfare. The main contributions
212 of this work are:

- 213 • The definition of a many-objective optimization model for rotational livestock graz-
214 ing that considers livestock fattening and their welfare.
- 215 • The definition of a set of objective functions that describe animal welfare.

216 This work is organized as follows. Section 2 introduces the assignment mathematical
217 model used in this work. Section 3 shows our approach through different case analyses in
218 meat production. After, Section 4 compares this work with previous work. Finally, Section
219 5 presents the conclusions and future works.

220 2 Our Approach

221 Rotational grazing involves dividing a farm into multiple paddocks, some of which are
222 grazed while others are left to rest [34] (see Figure 1). By reducing the total grazing area
223 and evenly distributing the cattle, this method ensures that forage is consumed uniformly,
224 making it possible to assign different herds to various paddocks [35].

225 An assignment problem, on the other hand, involves assigning resources to carry out
 226 tasks, with the aim of fulfilling specific goals such as maximizing benefits or minimizing
 227 costs [36–38]. Thus, the problem of rotational grazing can be viewed as an assignment
 228 problem, where the relationship between the resource and the task is equivalent to the
 229 correlation between the herds and paddocks in the assignment model.

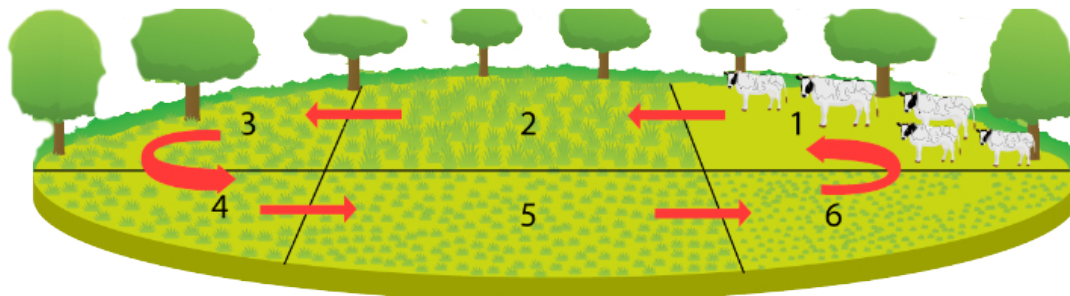


Figure 1: A graphical representation of a rotational grazing system (*Source: Own elaboration*).

230 This paper proposes a new approach to rotational livestock grazing that takes into
 231 consideration animal welfare by means of a mathematical model of many-objective opti-
 232 mization. Thus, what makes our approach novel are mainly two components: the proposal
 233 of indices to measure the animal welfare of cattle, which does not exist in the literature
 234 reviewed; and the proposal of an optimization model that in addition to maximizing animal
 235 weight gain, optimizes animal welfare by maximizing or minimizing the proposed welfare
 236 indices.

237 Specifically, we propose a dynamic optimization model for the daily allocation of animal
 238 lots to paddocks. The optimization is guided by six objectives associated with the weight
 239 gain of the animals, the walking distance of the cattle when they are moved from one
 240 paddock to another, and indices of their welfare such as food availability, temperature,
 241 noise, and space of each paddock. The optimization process consists of evaluating the
 242 conditions of each paddock on a daily basis and assigning cattle to paddocks in order to
 243 maximize cattle weight gain and animal welfare. Each optimization run takes into account
 244 the needs of each lot and calculates the estimated number of days each lot should remain in
 245 its respective paddock. The reasons for the proposals of welfare indices and the proposed
 246 mathematical model of rotational grazing are detailed below.

247 2.1 Animal welfare in our approach

248 The five animal freedoms are a set of principles that establish the necessary conditions for
 249 animal welfare [39]. Animal welfare freedoms consist of:

- 250 • Freedom from hunger and thirst: Continuous access to water and high-quality feed
 251 is fundamental to animal welfare.
- 252 • Freedom from discomfort: Prevention and treatment of discomfort are essential to
 253 ensure animal welfare.
- 254 • Freedom from pain, injury, and disease: Early detection and treatment of illness and
 255 injury are essential to ensure animal welfare.
- 256 • Freedom from fear and distress: Handling, transport, and slaughter of animals should
 257 be conducted in a manner that minimizes stress and distress to the animals.

- Freedom to express normal behavior: It is important to provide an environment that allows animals to express their natural behavior, such as foraging for food and water, moving freely, and socializing with other animals of their species.

These freedoms are fundamental to animal welfare, and their fulfillment is essential to ensure the health and well-being of animals. It also improves the quality of animal products for human consumption [40].

In this paper, we propose a mathematical optimization model that includes objectives aimed at increasing animal welfare by maximizing or minimizing variables that measure paddock conditions that are directly related to the freedoms described above. In addition to the weight gain and the distance traveled by the animals, it is proposed to assign herds to paddocks optimizing the following variables: the amount of available forage, noise level, temperature, and available space. The optimization of these conditions together allows for rotational grazing that, in addition to increasing the weight of the cattle, also seeks to improve animal welfare.

Each of these indicator variables is associated with one or more freedoms. For example, access to food helps animals not go hungry, i.e., the more food available, the less hungry the animals are, so the variable "Amount of Forage" is strongly and positively related to freedom from hunger and thirst. However, the amount of forage is also positively associated, albeit less strongly, with the freedom of cattle to express their normal behavior, which includes foraging for food and water. Additionally, access to feed allows the animal to eat properly and get the nutrients it needs, which decreases the risk of disease.

Table 1 shows the strength and direction (positive or negative) of the relationship between the proposed target variables and the freedoms that guarantee animal welfare. The variable Distance traveled is also included. Weight gain is not included in the table because it is related to animal mass gain and not to animal welfare.

Table 1: Relationship between objective variables and animal freedoms.

	Freedom from hunger and thirst	Freedom from discomfort	Freedom from pain	Freedom from fear and distress	Freedom to express normal behavior
Distance travelled		Moderate (-)	Weak (-)	Strong (-)	
Quantity of forage	Strong (+)		Weak (+)		Moderate (+)
Space		Moderate (+)		Weak (+)	Strong (+)
Noise level		Strong (-)	Weak (-)	Moderate (-)	
Temperature		Strong (-)	Weak (-)	Weak (-)	

A positive relationship (+) between the target variable and animal freedom indicates that the higher the value of the variable, the better the welfare condition of the animal. Therefore, the objective variables that have a positive relationship with the freedoms must be maximized and those with a negative relationship must be minimized. For example, if the noise level is too high, then it can generate discomfort in the animals, increase stress, cause distress, and even make them sick. Therefore, the variable "Noise level" has a negative relationship with the absence of discomfort, pain, fear, and distress. Thus, one of the objectives is to minimize the noise level. This is formalized mathematically in section 2.2, where the proposed mathematical optimization model is described.

292 In the literature reviewed, there are no parameters or criteria for measuring the animal
 293 welfare of cattle in grazing systems. Our work is the first to propose metrics to quantify
 294 the animal welfare of cattle.

295 It is good to recognize that the analysis that we have just done about animal freedoms
 296 and how to model them can lead to certain conflicts that will be analyzed in future works.
 297 For example, the transfer of a batch of cattle from one pasture to another motivated by the
 298 weight gain that the animals can acquire if the pasture to which they are transferred has
 299 better pasture conditions (quantity and quality), can lead to weight loss of the animal due
 300 to fat loss caused by walking and changing feeding places (can cause stress to the animal).
 301 Furthermore, the quantity and quality of forage in paddocks are not necessarily positively
 302 related to temperature, noise, or spaces, so a paddock with good forage conditions may
 303 also have very poor comfort conditions. Thus, it is possible that in some cases, the weight
 304 gain of livestock conflicts with animal welfare during the grazing process. That is why it
 305 is interesting to approach the problem of rotational grazing as a multi-objective problem
 306 that allows analyzing these objectives individually, in groups or globally, and add new ones
 307 that consider these possible conflicts.

308 2.2 Proposed many-objective optimization model

309 Let n and m be the total number of herds and the total number of paddocks in the grazing
 310 system, respectively. In real life, n is less than or equal to m . However, classically in
 311 operations research is assumed that an assignment model must always be balanced in
 312 order to be solved [41]. This assumption will be used in this work. Therefore, in the
 313 case where the number of herds is less than the number of paddocks, fictitious herds are
 314 virtually created in order to make n and m equal. When the model is implemented in
 315 real life, then the paddocks with fictitious herds assigned are empty paddocks. Thus, the
 316 mathematical formulation is based on the assumption that the system is balanced and that
 317 rotational grazing is performed for p days. Then, the binary decision variable x_{ij}^t is defined
 318 to indicate if the herd i is assigned to the paddock j at time t (days), with $i, j = 1, 2, \dots, n$
 319 and $t = 1, 2, \dots, p$.

320 This paper proposes a many-objective optimization model composed of six objectives
 321 corresponding to the weight gain and movements of the animals, and to the five animal
 322 freedoms. The first objective is to maximize the total weight gain of the animals due to the
 323 allocation of the flocks to paddocks at each time t . The mathematical function representing
 324 this objective is given by equation 1.

$$325 \text{ Maximise } Z_1 = \sum_{i=1}^n \sum_{j=1}^n G_{ij}^t x_{ij}^t \quad (1)$$

326 where G_{ij}^t is the weight gain to be obtained by herd i in paddock j estimated at time t .

327 The second objective is to minimize the total distance traveled by the animals when
 328 moving from one paddock to another each time they are moved during the defined rotational
 329 grazing period, which can be three months or one year, for example.

330 The mathematical function is given by equation 2, in which D_{ij}^t is the distance in
 331 meters between paddocks i and j at a time t to move herd k between these paddocks.

$$332 \text{ Minimise } Z_2 = \sum_{k=1}^n \sum_{i=1}^n \sum_{j=1}^n D_{ij}^t x_{ki}^t x_{kj}^{t-1} \quad (2)$$

333 For animal welfare, Table 2 describes the mathematical notation used for objective
 variables representing the levels of animal freedom. Since it is desired to maximize the

334 available amount of food and space but to minimize noise and temperature levels, then the
 335 mathematical functions for these objectives are given by equations 3-6.

Table 2: Animal welfare index variables.

Variables	Description
FI_{ij}^t	Forage Index of allocation of herd i to paddock j at time t .
SI_{ij}^t	Space Index of allocation of herd i to paddock j at time t .
NI_{ij}^t	Noise Index of allocation of herd i to paddock j at time t .
TI_{ij}^t	Temperature Index of allocation of herd i to paddock j at time t .

$$\text{Maximise } Z_3 = \sum_{i=1}^n \sum_{j=1}^n FI_{ij}^t x_{ij}^t \quad (3)$$

$$\text{Maximise } Z_4 = \sum_{i=1}^n \sum_{j=1}^n SI_{ij}^t x_{ij}^t \quad (4)$$

$$\text{Minimise } Z_5 = \sum_{i=1}^n \sum_{j=1}^n NI_{ij}^t x_{ij}^t \quad (5)$$

$$\text{Minimise } Z_6 = \sum_{i=1}^n \sum_{j=1}^n TI_{ij}^t x_{ij}^t \quad (6)$$

336 The amount of forage available within a paddock j at a time t does not depend on the
 337 herds assigned to it. However, it is important to take into account the nutritional needs of
 338 the animals when assigning a herd to a paddock since it influences the amount of weight
 339 the animals can gain. Since nutritional need depends directly on the weight of the animal,
 340 then we propose to calculate the forage index by means of the expression 7, which measures
 341 the amount of forage (in mass units) available per unit of weight (in mass units) of the
 342 herds of animals. In other words, this index indicates the amount of forage available per
 343 unit of weight of cattle

$$FI_{ij}^t = \frac{TF_j^t}{W_i^t}, \quad \forall i, \forall j, \forall t \quad (7)$$

344 where TF_j^t is the total amount of forage within paddock j at time t , and W_i^t is the total
 345 weight of the animals in herds i at time t .

346 For the space index, it is necessary to take into account the space occupied by the herd,
 347 which depends on the size of the animals, which in turn is directly related to the weight.
 348 Thus, denoting the area of the paddock j as A_j , the space index is calculated with the
 349 expression 8, which represents the amount of space available per unit weight of livestock.

$$SI_{ij}^t = \frac{A_j^t}{W_i^t}, \quad \forall i, \forall j, \forall t \quad (8)$$

350 On the other hand, we consider that the noise and temperature sensation experienced
 351 by the animals is positively related to their size and to the number of animals in the herds.
 352 Therefore, as a first approximation to the measurement of noise level and temperature
 353 indices of the allocation of a herd i to a paddock j , we propose the equations 9 and 10,
 354 where N_j^t and T_j^t are the noise and temperature levels of paddock j at time t , respectively.
 355 They indicate the noise level and temperature level of each paddock boosted by the stocking

356 rate (total weight) of each lot. Thus, taking the noise index as an example, the assignment
 357 of a specific lot of cattle to a specific paddock has an associated noise index that corresponds
 358 to the noise level of the paddock boosted by the stocking rate of the lot.

$$NI_{ij}^t = N_j^t \cdot W_i^t, \quad \forall i, \forall j, \forall t \quad (9)$$

$$TI_{ij}^t = T_j^t \cdot W_i^t, \quad \forall i, \forall j, \forall t \quad (10)$$

359 The parameters $TF_j^t, W_i^t, A_j, N_j^t$ and T_j^t are read from system information or estimated
 360 at time t .

361

362 On the other hand, defining O_{ij}^t as the estimated occupancy time (in days) at time t
 363 that a herd i must remain in paddock j to consume the total quality forage, setting g_j^t as
 364 the average daily weight gain of an animal in paddock j (influences the type of pasture in
 365 the paddock) on the day t (influences the time of year), and defining C_i^t as the number
 366 of cattle in the herd i at time t , the total weight gain obtained by a herd of animals if
 367 assigned to a given paddock is calculated by the expression:

$$G_{ij}^t = O_{ij}^t \cdot g_j^t \cdot C_i^t, \quad \forall i, \forall j, \forall t \quad (11)$$

368 The occupancy time of herds in the paddock is calculated using the expression 12, where
 369 QF_j^t is the amount of quality forage in paddock j at time t and NR is the daily nutritional
 370 requirement of an animal expressed as a fraction of its weight, with $0 \leq NR \leq 1$.

$$O_{ij}^t = \frac{QF_j^t}{NR \cdot W_i^t}, \quad \forall i, \forall j, \forall t \quad (12)$$

371 The total area of each paddock is an important constraint when making daily alloca-
 372 tions. Defining a_i^t as the estimated area of occupancy of the herds i at time t , the inequality
 373 13 must be satisfied.

$$a_i^t \cdot x_{ij}^t \leq A_j, \quad \forall i, \forall j, \forall t \quad (13)$$

374 At any time t , each herd must be assigned to a single paddock and each paddock must
 375 be assigned to a single herd. These restrictions are represented by the expressions 14 and
 376 15.

$$\sum_{j=1}^n x_{ij}^t = 1, \quad \forall i, \forall t \quad (14)$$

$$\sum_{i=1}^n x_{ij}^t = 1, \quad \forall j, \forall t \quad (15)$$

377 Finally, equation 16 expresses the constraint corresponding to the binary nature of the
 378 decision variable

$$x_{ij}^t \in \{0, 1\}, \quad \forall i, \forall j, \forall t \quad (16)$$

379 Since the proposed model considers the evolution of parameters and variables over
 380 time, it is a dynamic optimization model. The model must be run daily after updating
 381 the information corresponding to the characteristics of the paddocks and cattle herds, such
 382 as total forage quantity, forage quality, noise and temperature levels, and animal weight,
 383 among other parameters. On each day, an efficient solution to the model is found, which
 384 allows an efficient allocation of herds to paddocks, seeking to maximize weight gain but
 385 taking into account animal welfare. In this way, depending on the values of the target

386 variables (weight gain and animal welfare indexes), the decision is made to assign each lot
387 of cattle to a specific paddock. The analysis of the proposed model is presented in section
388 3.

389 3 Model Evaluation

390 The effectiveness of the optimization model proposed in this work was analyzed through
391 a simulation of a rotational grazing system. The characteristics of the simulation study
392 conducted are described below.

393 3.1 Description of Simulation Study

394 A simulation of a 90-day rotational cattle grazing system was run to evaluate the perfor-
395 mance of the proposed mathematical model. Two types of grazing systems are considered,
396 a traditional grazing system that does not use mathematical optimization and a grazing
397 system that uses the optimization model proposed in this work.

398 Before starting the simulation, parameter values such as the number of cattle herds and
399 the number of paddocks are defined. The characteristics of the paddocks such as location
400 within the farm, area, type of pasture, the amount of forage, and noise and temperature
401 levels, are randomly generated. Likewise, in the case of cattle herds, characteristics such
402 as gender, weight, and age of each animal are randomly produced. For the daily growth
403 of the pasture, the influence of the season of the year, the species of the pasture, and
404 its flowering time were taken into account. **On the other hand, at the beginning of the
405 simulation, the farm begins by having all its paddocks with a complete and known quantity
406 of forage, which is made up of quality forage and non-quality forage. In turn, on each day
407 of the simulation, the amount of total forage changes depending on the amount of forage
408 consumed by the livestock, the natural growth of the grass, and the time of year (rainy or
409 dry seasons).**

410 Quality forage corresponds to the part of the pasture that provides the greatest weight
411 gain to the animals due to its nutrients, has the best flavor, and is found in the upper part
412 of the plant. Because of this, **the simulation assumes that** quality forage is the first thing
413 that animals consume, and therefore, is the first to be depleted during grazing. When the
414 quality forage runs out, the animal proceeds to consume the rest of the forage.

415 **The weight of the animals is updated at the end of each day based on the quantity
416 and quality of forage consumed, their age, gender, weight, and distance traveled when
417 moving from one paddock to another.** The parameters that are defined before starting the
418 simulation are presented in Table 3.

419 The *output variables* are: (1) Final weight of animals, (2) Average weight of animals,
420 (3) Average weight gain of the animals, (4) Final forage of each paddock (quality and
421 non-quality) (5) Average forage (quality and non-quality)

422 In summary, the discrete event simulator macro-algorithm of the cattle rotation system
423 is shown in Figure 2. The simulator is located at [https://github.com/devraxielh/
424 Simulador_Ganadero](https://github.com/devraxielh/Simulador_Ganadero).

Table 3: Parameter identification

	Parameter
General	Number of days to be simulated
	Number of paddocks
Paddocks	Daily growth rate of the pasture (in percentage units)
	Plant species
	Rate of extra increase in the rainy season
	Rate of loss in the dry season
	Rate of loss due to flowering
	Daily weight gain of an animal depending on the quality of the forage
	Minimum and maximum area of a paddock
	Minimum and maximum capacity of the paddocks at the beginning of the simulation
	The measurements of the farm within which the paddocks are randomly located before starting the simulation
	Initial fraction of the total forage that is quality forage
	Amount of forage per square meter that grows in a paddock on a rest day after the capacity reaches zero
	Number of paddocks
	Area of each paddock (m^2)
	Location of the paddocks within the farm
	Forage of each paddock (kg)
	Number of consecutive days of occupation allowed per paddock
Number of days that a paddock must remain unoccupied after the maximum number of consecutive days of occupation allowed	
Herds	Number of herds
	Nutritional requirement, as a percentage of the total weight of a cattle herd that the herd needs to consume daily to increment the weight
	Minimum and maximum number of animals per herd
	Weight loss per walk (kg/m)
	Daily nutritional requirement of an animal

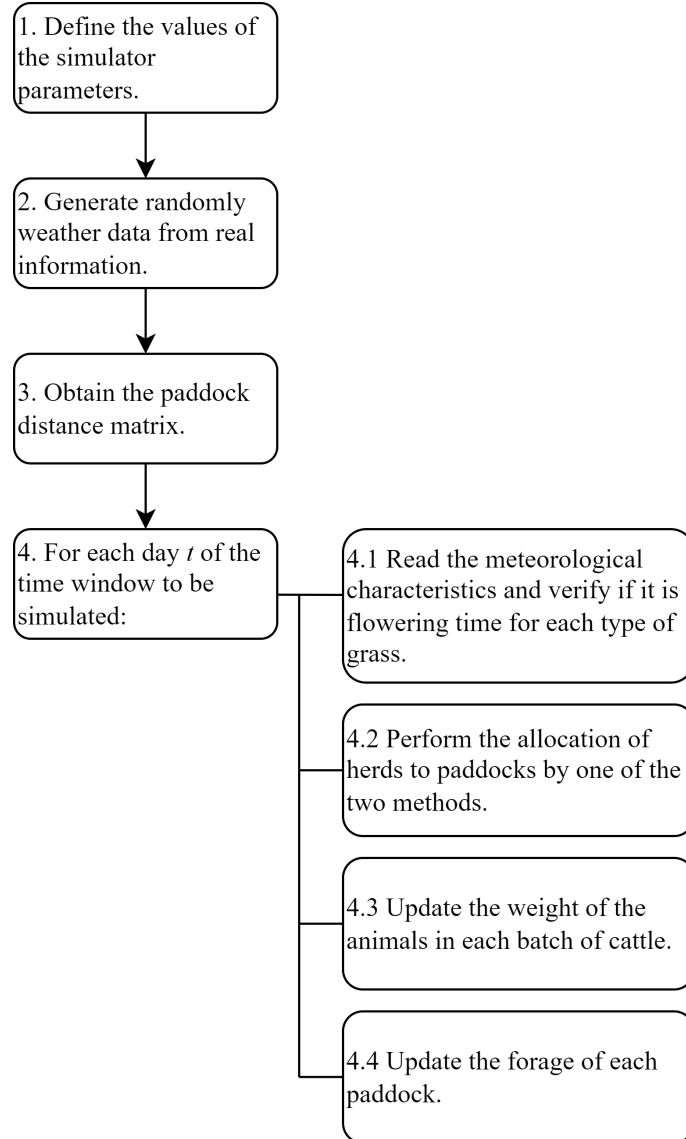


Figure 2: General simulation algorithm flowchart (*Source: Own elaboration*).

425 The simulation was programmed in R software due to its potential in statistical data
 426 analysis but was connected to Python to make use of the Platypus library in which several
 427 algorithms are available for the solution of multi-objective and many-objective optimiza-
 428 tion problems. Since the objective of this work is to innovate in the way of analyzing
 429 the rotational grazing problem by including additional objectives for livestock weight gain
 430 (classical approach), it is not of interest to compare the performance of algorithms for
 431 solving multi-objective optimization models nor to propose a particular heuristic for the
 432 solution of the proposed model. Therefore, the NSGA-III evolutionary algorithm was used
 433 to solve the optimization model because of its good performance in multi-objective opti-
 434 mization problems according to the literature [20, 21, 38]. Particularly, the computational
 435 complexity of the NSGA-III algorithm is $O(n_g n_o n_p^2)$, where n_g is the number of gener-
 436 ations, n_o is the number of objectives, and n_p is the population size, but in turn, the
 437 objective functions, in our case, depend on the number of livestock herds and paddock.
 438 On the other hand, a grid search was carried out to adjust the hyperparameters of the
 439 evolutionary algorithm, with which it was determined to use a population of 10 individuals
 440 and 10,000 runs, among other optimized parameters.

441 3.2 Experimental Design

442 The validation of the proposed optimization model is carried out by means of an experi-
 443 mental design considering three factors: the number of herds/lots, the number of animals
 444 per herd and the grazing strategy. The levels of each factor are presented in Table 4. Thus,
 445 we have an experimental design with $3 \times 3 \times 2 = 18$ treatments, for each of which 6 simulation
 446 runs were executed.

Table 4: Factors and levels of the experimental design.

Factor	Level
Number of herds	1, 4, 15
Number of animals per herd	2, 10, 50
Grazing strategy	Traditional Rotational grazing, Rotational grazing using our optimization model.

447 The traditional rotational grazing strategy consists of a grazing system in which animal
 448 lots are periodically rotated within the farm taking into account the number of days that
 449 each paddock must remain unoccupied for pasture recovery, the estimated forage of the
 450 unoccupied paddocks, the distances between paddocks that the animals must travel, the
 451 area of the paddocks and the size of the lots (number of animals and their weight). For
 452 example, on some farms, it is decided by default that animal lots remain in each paddock
 453 to which they are assigned for 30 consecutive days. After this time, it is decided to move
 454 the lot to a paddock with the largest amount of forage and as close as possible to simplify
 455 the process of transporting the animals. In general, the allocation of lots to paddocks
 456 is based on the perception of the decision-maker, is not guided by a formal optimization
 457 strategy, and does not take into account animal welfare.

458 On the other hand, the rotational grazing strategy using the optimization model is
 459 based on a daily execution of the mathematical model after reading or calculating the levels
 460 of the system state variables such as animal weight, paddock forage, location of the cattle
 461 herds, etc. The proposed model is solved using a many-objective optimization problem-
 462 solving algorithm. The algorithm finds a set of effective solutions called the Pareto front.
 463 Since the priority is the weight gain of the animals, the effective solution of the Pareto
 464 front that has the highest value in the objective variable Total Weight Gain is selected as
 465 the best solution. Based on this selected solution, an allocation of lots to paddocks is made
 466 to optimize the weight gain of the animals and to take care of animal welfare. Depending
 467 on the allocation obtained by the model, some lots remain in the paddock where they
 468 are located and others are moved to another paddock. Then, the system state variables
 469 are updated. Nevertheless, if higher priority is given to animal welfare, then the solution
 470 chosen as the best would mean a different allocation. Thus, depending on the order of
 471 priority given to the objectives, different allocations of lots to paddocks can be obtained.

472 **Regarding water for livestock, according to experts, the usual is that in the design of**
 473 **the pastures, farmers ensure that they provide the necessary water to the animals in each**
 474 **of them so that the animals can satisfy this need at the time they require it. Thus, in**
 475 **the simulation process it is assumed that on the farm where rotational grazing is carried**
 476 **out, the animals have access to sufficient water to satisfy their needs in any pasture.**
 477 **Therefore, this work does not include parameters or variables related to water availability**
 478 **or consumption.** The rest of the simulation parameters are the same for all the design
 479 treatments (combinations of factor levels), and their values are presented in Table 5. The
 480 selection of these parameters was defined with the advice of farmers and zootechnical
 481 professionals from Finca El Rosario (Montería, Colombia), who are experts in rotational
 482 grazing of cattle in the Colombian tropics. Several consultation meetings were held with

483 these experts in which it was determined that these parameters are the most influential in
 484 the rotational grazing process according to their experience.

485 Particularly, forage quality has a great impact on cattle weight gain [42]. Now, the
 486 amount of quality forage in the pasture depends on factors such as type of grass, proportion
 487 of young leaves [43], height of the plant [44], or season of the year [45], among others. Thus,
 488 to simulate the positive impact that quality forage has on the weight gain of livestock, the
 489 increase in the animal's weight gain when consuming quality forage with respect to the
 490 consumption of non-quality forage was assumed to be a higher percentage than varies
 491 between 10% and 25%, depending on the type of grass and the season of the year. These
 492 values were suggested by the consulted experts, who considered them reasonable values
 493 based on their experience in the behavior of grasses used in the Colombian tropics.

494 Following the procedure described in subsection 3.1 and the guidelines in subsection
 495 3.2, the experiments carried out in this work are easily reproducible, and allow the addition
 496 of new variables or factors that can be considered important or influential in rotational
 497 cattle grazing.

Table 5: Simulation parameters

Parameter	Value
Simulated rotational grazing days	90
Number of paddocks	30
Minimum area of a paddock (m^2)	45000
Maximum area of a paddock (m^2)	55000
Minimum capacity of a paddock (kg of grass)	3000
Maximum capacity of a paddock (kg of grass)	3500
Minimum noise level (decibels)	30
Maximum noise level (decibels)	80
Minimum temperature (degrees Celsius)	30
Maximum temperature (degrees Celsius)	45
Maximum number of consecutive days a paddock can be occupied consecutively.	3
Ideal number of days a paddock should remain unoccupied after being used.	25
Forage (kg/m^2) that grows in one day in a paddock after it has been completely consumed.	0.08
Fraction of total forage that is quality forage	0.3
Minimum initial weight of an animal (kg)	370
Maximum initial weight of an animal (kg)	530
Weight loss per walk (kg/m)	0.00001
Fraction of weight gain that is in addition to the average gain for quality forage	0.15
Daily nutritional requirement of an animal (percent of its weight)	11%
Prime rate of daily growth of grass (forage)	12%
Increase in forage due to rainfall gain	12%
Decrease in forage due to drought loss	4%
Decrease in forage due to flowering loss	3%

498 **3.3 Experimental Results**

499 Since the main objective of interest is to maximize animal weight gain, the model perfor-
 500 mance metric used in the experimentation is the average animal weight-gain (AWG), which
 501 is useful for comparing the two grazing strategies considered in the simulation study. The
 502 AWG allows measuring the average amount of weight gained by the animals due to graz-
 503 ing during the study time since it calculates the average weight difference of the animals
 504 between the last day of grazing and the first day of grazing. The AWG is calculated as
 505 follows:

$$AWG = \frac{1}{N} \left(\sum_{k=1}^N W_{fk} - \sum_{k=1}^N W_{0k} \right) \quad (17)$$

506 where W_{0k} and W_{fk} are the weights of the animal k at the beginning and end of the sim-
 507 ulation, respectively, and N is the total number of animals.

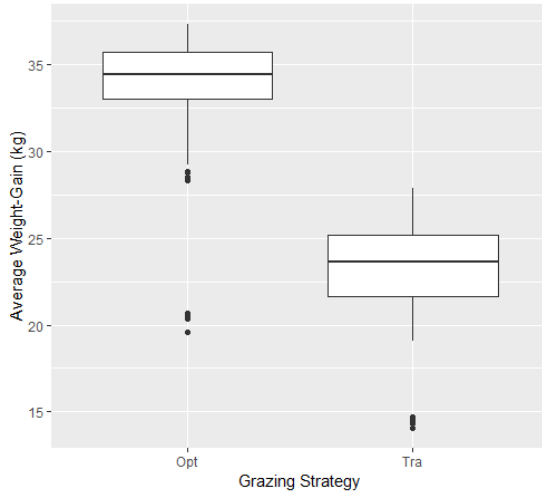
508
 509 Figure 3-(a) shows the box plots of the AWG obtained for the two grazing strategies
 510 evaluated without discriminating the number of cattle lots or the number of animals per lot.
 511 According to the diagrams, in general, the grazing strategy using the proposed optimization
 512 model (Opt) achieves an average weight gain (with a mean of 32.73 kg and standard
 513 deviation of 4.9 kg), higher than the traditional grazing strategy (Tra) (with a mean of
 514 22.82 kg and standard deviation of 3.7). However, a greater presence of outlier data is also
 515 observed in the grazing strategy with optimization, specifically in the lower tail, indicating
 516 greater variability.

517 However, it is necessary to compare the performance of the rotational grazing strategy
 518 using the optimization model with traditional rotational grazing in different scenarios.
 519 Figures 3-(b), 3-(c) and 3-(d) show the AWG box plots of each grazing strategy for the
 520 simulated scenarios, where H1, H4 and H15, represent the cases of 1 herd, 4 herds and 15
 521 herds, respectively, and A2, A10, and A50 denote the cases of 2 animals, 10 animals and 50
 522 animals per herd, respectively. It is observed that in each of the scenarios considered in the
 523 experimental design, the optimization model produces higher AWG values than traditional
 524 grazing, showing superior performance in the task of generating animal weight gain. The
 525 arithmetic mean and standard deviation of the AWGs (in kg) of the simulation runs are
 526 presented in Table 6. For cases where the number of herds is 1 or 15, a decreasing trend
 527 in the mean AWG is observed as the number of animals increases. This is an expected
 528 result since an increase in flock size has a negative impact on feed availability, so animals
 529 consume less feed and gain less weight.

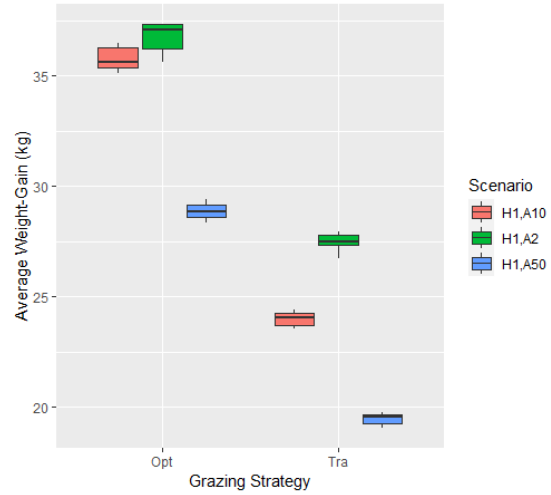
Table 6: Mean and standard deviation of AWGs (in kg) of the simulation replicates.

Number of herds	Number of animals per herd	Opt	Tra
1 herds	2 animals	36.73 (0.75)	27.46 (0.43)
	10 animals	35.75 (0.58)	23.98 (0.35)
	50 animals	28.85 (0.40)	19.46 (0.28)
4 herds	2 animals	34.84 (0.66)	26.19 (0.68)
	10 animals	35.63 (0.60)	21.97 (0.63)
	50 animals	33.43 (0.40)	24.88 (0.47)
15 herds	2 animals	35.19 (0.94)	23.22 (0.87)
	10 animals	33.85 (0.52)	23.79 (0.62)
	50 animals	20.38 (0.43)	14.42 (0.21)

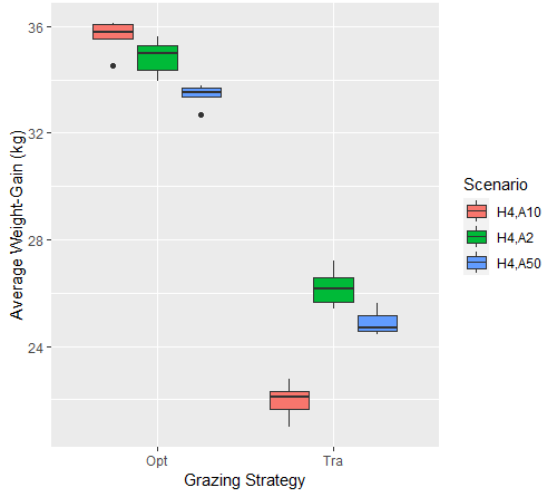
530 According to these results, the proposed optimization model for rotational grazing



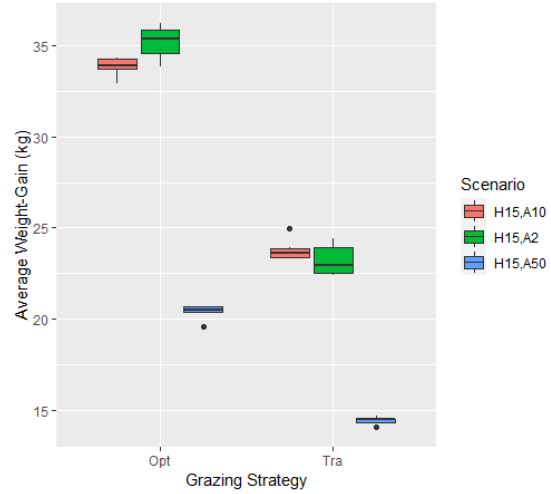
(a) General boxplot.



(b) Herds: 1; Animals: 2,10,50.



(c) Herds: 4; Animals: 2,10,50.



(d) Herds: 15; Animals: 2,10,50.

Figure 3: Boxplots of the AWG obtained for the grazing strategies for each scenario (Source: Own elaboration based on simulation results).

531 presents on average a higher average weight gain than that achieved by rotational grazing
 532 performed in the traditional way. The statistical verification of these assertions is presented
 533 in section 3.5.

534 3.4 Discussion about the Obtained Pareto Front

535 It is possible to find an optimal solution in single-objective optimization problems, but
 536 in the case of multi-objective problems, it is not possible to determine a single optimal
 537 solution for all the objectives because they are in conflict, i.e., improving one of them
 538 implies making others worse [20, 21, 38]. This situation justifies the concept of the Pareto
 539 front [15], which is the set of optimal solutions with the best compromises between the
 540 different objective functions.

541 On the other hand, heatmaps are frequently used for visualizing the objectives of a
 542 multi-objective problem with respect to individual solutions. They show the interaction
 543 between these two elements as a color of varying intensity. Thus, the heatmaps provide a
 544 2D visualization of how the objectives interact for any solution as well as how each objective

545 interacts with a given solution. Figure 4 shows the heatmaps of the six objectives (each
 546 column represents a goal: Z_1 = Weight gain; Z_2 = Distance traveled (Dist); Z_3 = Food
 547 (Forage), Z_4 = Space, Z_5 = Noise and Z_6 = Temperature (Temp)) and the solutions in
 548 our Pareto front. The variation in color intensity provides a clear visual cue on how the
 549 variables vary with respect to each other in each solution. Specifically, Figure 4 shows the
 550 heatmaps for the Pareto optimal points for one of the scenarios of our problem (15 herds
 551 and 10 animals) for different simulation days. This method allows the visualization of the
 552 behavior of the objectives in each Pareto solution.

553 The results show that with more days of simulation, solutions begin to prevail in the
 554 Pareto Front where the profit objective is the most relevant (see Figure 4.c). Thus, it is
 555 possible to stand out that with more days of simulation, solutions are achieved on the
 556 Pareto front that greatly degrade animal welfare goals. Figure 4.c shows that the weight
 557 is one of the more relevant variables (more intensive color in many solutions). Also, the
 558 Pareto solutions that are in the lower part of Figure 4.c combine with good values the
 559 objectives of animal welfare, but it is seen that for this, they degrade the goal of weight
 560 gain. In Figure 4.c, there are also solutions where all the objectives are degraded, and
 561 the only one that prevails with a good value is weight gain. In general, improving that
 562 objective may imply a worsening of animal welfare. But it is possible to achieve solutions
 563 that improve that objective without degrading those of animal welfare (for example, see
 564 solutions from the middle to the top of Figure 4.c).

565 On the other hand, it is possible to see that there is at least one Pareto solution where
 566 each objective reaches its best value (more intense color). No solutions are found that
 567 successfully achieving an animal welfare objective, degrades the rest of the animal welfare
 568 objectives (they are compatible with each other). **In summary, in this analysis of the Pareto
 569 front is observed that the greater the number of days of grazing, the weight gain objective
 570 becomes more relevant. That is, the longer the grazing time, there are more solutions on
 571 the Pareto front where weight gain becomes more important than animal welfare. On the
 572 other hand, we see that welfare objectives do not degrade each other. In other words, the
 573 weight gain goal is in conflict with the animal welfare goals, while the latter are not in
 574 conflict with each other.**

575 This type of analysis can help decision makers find an appropriate solution from the
 576 Pareto-optimal set. Finally, the most suitable solution will be obtained considering aspects
 577 of the environment/business, such as the current conditions of the farm, the meat market,
 578 and possible future improvements in each of them, among other things.

579 3.5 Quality analysis

580 To test statistically whether there are significant differences in AWG between treatments
 581 (simulation scenarios), an effects model is fitted with the results of the experimental design
 582 described in section 1. With such a model for the analysis of variance, we intend to model
 583 linearly the effects that the combinations of simulation scenarios have on the weight gain
 584 metric. Thus, the model of the effects is given by:

$$AWG_{ijk_r} = \mu + h_i + a_j + s_k + (ha)_{ij} + (hs)_{ik} + (as)_{jk} + (has)_{ijk} + \varepsilon_{ijk_r}, \quad (18)$$

585 where AWG_{ijk_r} is the average animal weight-gain of the ijk_r -th observation, r the index
 586 of the simulation replicate, μ the overall average effect, h_i the effect of the i -th level of
 587 the Number of herds factor, a_j the effect of the j -th level of the factor Number of animals
 588 per herd, s_k the effect of the k -th level of the factor Grazing strategy, $(ha)_{ij}$ the effect of
 589 the interaction between h_i and a_j , $(hs)_{ik}$ the effect of the interaction between h_i and s_k ,
 590 $(as)_{jk}$ the effect of the interaction between a_j and s_k , $(has)_{ijk}$ the effect of the interaction

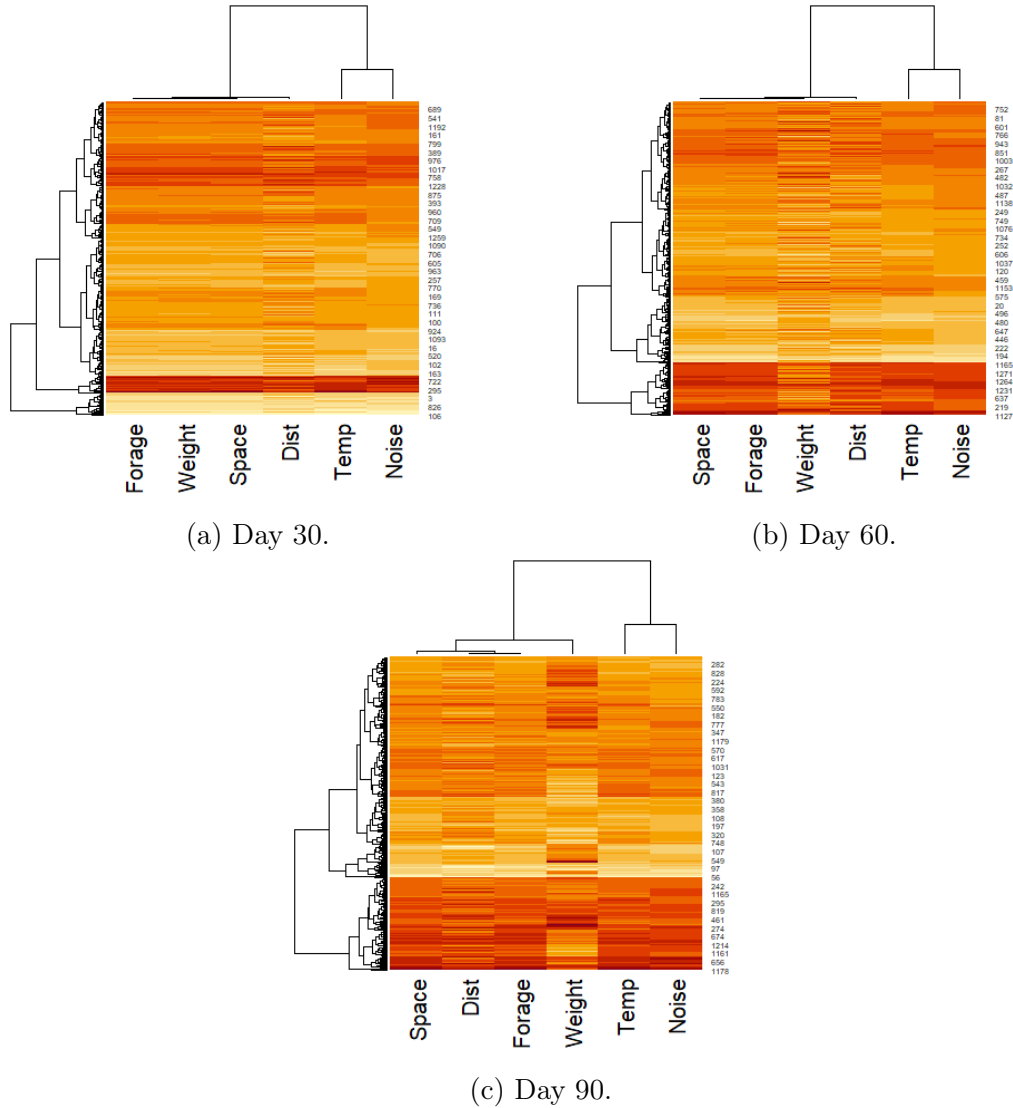


Figure 4: Heatmap of the 6 objectives in the solutions on the Pareto front for the scenario of 15 herds and 10 animals (*Source*: Own elaboration based on simulation results).

591 between h_i , a_j and s_k , with $i = 1, 2, 3$; $j = 1, 2, 3$; $k = 1, 2$ and $r = 1, 2, \dots, 6$.

592

593 Note: The significance level for the hypothesis testing performed in this section is
 594 $\alpha = 0.01$.

595 3.5.1 Statistical verification of the optimization model

596 To be confident in the analysis of variance, it is necessary that the assumptions of the
 597 statistical model, which correspond to independence, normality, and homogeneity of vari-
 598 ance of the errors, are met. In Figure 5, are presented: the plot of the residuals in their
 599 time order (a), the histogram of the residuals (b), and the plot of the residuals against
 600 the fitted values of the response variable (c). In Figure 5 (a) there is no increasing or
 601 decreasing trend in the values of the residuals over time, moreover, the dispersion remains
 602 stable. Therefore, it is suspected that the errors are independent. On the other hand,
 603 the histogram (5 (b)) shows a clear bell shape with great symmetry, but a disturbance
 604 is observed in the left tail of the distribution. Thus, it appears that the errors possess a

605 Normal distribution, but this needs to be confirmed. As for the homogeneity of variances,
 606 in Figure 5 (c), the variability of the residuals is not shown to be stable, which suggests
 607 that the homoscedasticity assumption is not met. In summary, Figure 5 indicates that the
 608 errors are independent, normally distributed with homogeneous variance.

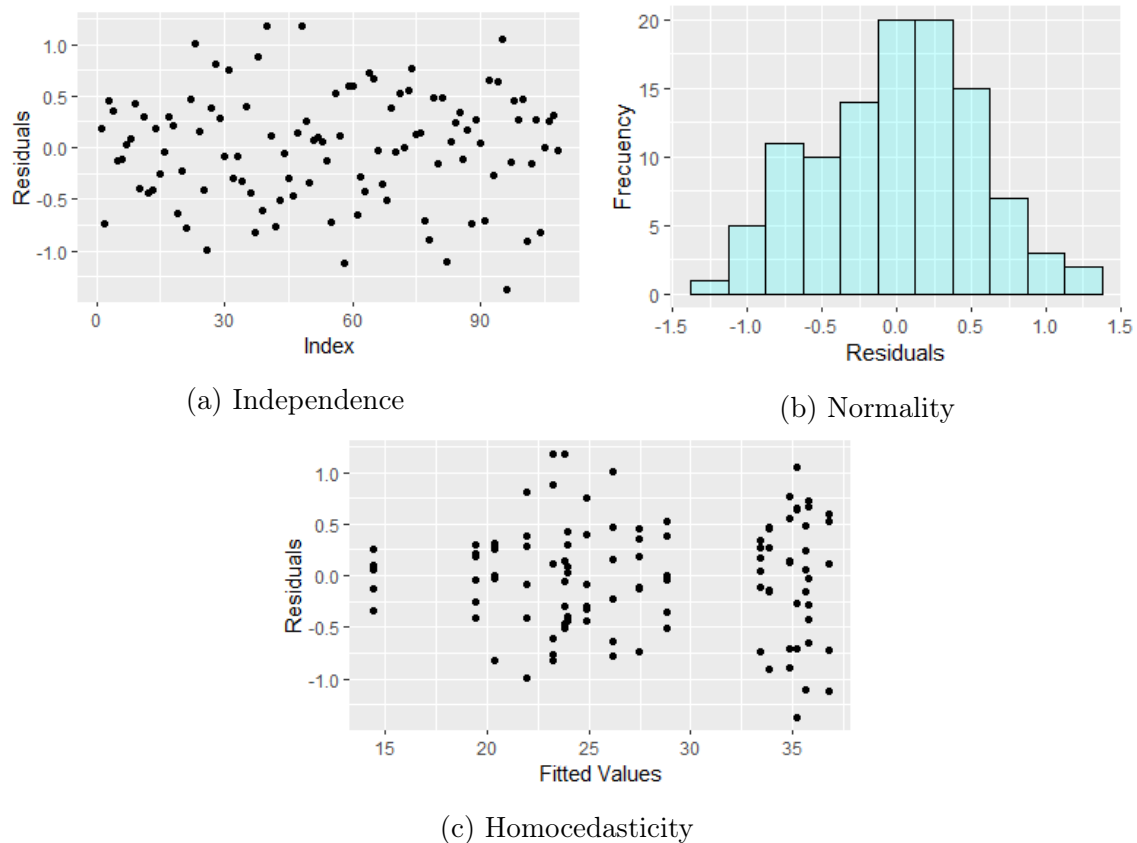


Figure 5: Validation of ANOVA assumptions (*Source: Own elaboration based on simulation results*).

609 To formally validate compliance with the assumptions of independence, normality, and
 610 homogeneity of variances, the Durbin-Watson, Shapiro-Wilk and Bartlett statistical tests
 611 were performed, respectively. The p-values obtained when performing the tests were 0.3432,
 612 0.7549, and 0.2397, respectively, which are greater than the significance level $\alpha = 0.01$
 613 previously defined. Therefore, it is formally verified that the effects model meets the
 614 assumptions and the Analysis of Variance can proceed.

615 3.5.2 Analysis of variance (ANOVA)

616 Table 7 presents the hypotheses tested in the analysis of variance and their respective P-
 617 values, which are all less than the 0.01 significance level. This indicates that all the null
 618 hypotheses are rejected so that sufficient statistical evidence was found to affirm that there
 619 are differences between the effects of the levels of the factors on the mean weight gain of
 620 the animals. In particular, the rejection of hypothesis number 3, which corresponds to
 621 the comparison of the effects between grazing strategies, shows that there are significant
 622 differences between the effects of traditional rotational grazing and rotational grazing based
 623 on the proposed optimization model.

Table 7: ANOVA hypothesis testing results of the model.

Hypothesis H_0	Description of hypothesis	P-Value
$h_i = 0, \forall i$	The effects of the levels of the Number of herds factor are equal to zero.	$< 2.2e - 16$
$a_j = 0, \forall j$	The effects of the levels of the Number of animals in the herds are equal to zero.	$< 2.2e - 16$
$s_k = 0, \forall k$	The effects of the levels of the Grazing strategy are equal to zero.	$< 2.2e - 16$
$(ha)_{ij} = 0, \forall i, j$	There is no interaction between the Number of herds and the Number of animals in the herds.	$< 2.2e - 16$
$(hs)_{ik} = 0, \forall i, k$	There is no interaction between the Number of herds and the Grazing strategy.	0.001357
$(as)_{jk} = 0, \forall j, k$	There is no interaction between the Number of animals in the herds and the Grazing strategy.	$< 2.2e - 16$
$(has)_{ijk} = 0, \forall i, j, k$	There is no interaction between the Number of herds, the Number of animals in the herds and Grazing strategy.	$< 2.2e - 16$

624 Since it was found that there are significant differences between the effects of the
625 factor levels and that there is interaction between some of them, multiple comparison tests
626 are performed. In this case, the Tukey HSD (Honestly Significant Difference) test was
627 performed, and the results are presented in Table 8. The third column of the table shows
628 the groups of means resulting from the Tukey test. If two scenarios have the same letter,
629 it signifies that the means of the AWGs are statistically equal. For example, scenarios 1
630 and 2 belong to the **a** group, so there is no significant difference in the AWG means. The
631 same is true for scenarios 2 and 4, which belong to the **b** group, but scenarios 1 and 4
632 do not share any letters, then their AWG means have significant differences. Since the
633 scenarios are ordered in descending order according to the value of the mean AWG, it is
634 observed that the use of the optimization model generates a significantly higher weight
635 gain than that achieved without using it in any scenario. These results show that the
636 rotational grazing strategy using the proposed optimization model produces a statistically
637 higher mean weight gain than the traditional grazing strategy.

Table 8: Results of Tukey HSD test.

N°	Scenario	\overline{AWG} (kg)	Group
1	H1, A2, Opt	36.73	a
2	H1, A10, Opt	35.75	ab
3	H4, A10, Opt	35.64	ab
4	H15, A2, Opt	35.19	b
5	H4, A2, Opt	34.84	bc
6	H15, A10, Opt	33.85	cd
7	H4, A50, Opt	33.43	d
8	H1, A50, Opt	28.86	e
9	H1, A2, Tra	27.46	f
10	H4, A2, Tra	26.19	g
11	H4, A50, Tra	24.88	h
12	H1, A10, Tra	23.98	hi
13	H15, A10, Tra	23.79	hi
14	H15, A2, Tra	23.22	i

15	H4, A10, Tra	21.97	j
16	H15, A50, Opt	20.38	k
17	H1, A50, Tra	19.46	k
18	H15, A50, Tra	14.42	l

638 4 Comparison with Previous Works

639 In this section, we propose several criteria to compare previous studies related to animal
640 grazing optimization with our approach. These criteria are:

- 641 • **Criterion 1:** The study proposes a mathematical optimization model applied to
642 Precision farming processes.
- 643 • **Criterion 2:** The study approaches the rotational grazing problem by means of an
644 optimization model using many objectives.
- 645 • **Criterion 3:** The study takes into account the welfare of animals through their
646 freedoms.

647 Criterion 1 is relevant because it allows addressing the problem in a quantitative and
648 systematic way, using advanced tools and techniques of Precision farming to find optimal
649 solutions. Criterion 2 is important because rotational grazing involves managing multiple
650 variables and objectives, such as maximizing livestock weight and optimizing pasture uti-
651 lization. A multi-objective approach allows these different aspects to be considered and
652 balanced more effectively, helping farmers take actions that benefit both the productivity
653 and sustainability of the system. Finally, criterion 3 is critical because animal welfare is
654 an increasingly important aspect of livestock production. Consideration of animal free-
655 doms, such as the freedom to move, behave naturally, and avoid stressful situations, can
656 significantly improve the living conditions of animals. The integration of these criteria
657 allows finding solutions that promote both productivity and animal welfare. In Table 9, a
658 qualitative comparison with related studies is made, based on previous criteria.

Table 9: Comparison with previous works.

	Criterion 1	Criterion 2	Criterion 3
[11]	✓	✗	✗
[12]	✓	✗	✗
[13]	✓	✗	✗
[14]	✓	✗	✗
[15]	✓	✗	✗
[16]	✓	✗	✗
[17]	✓	✗	✗
[19]	✓	✗	✗
[18]	✓	✗	✗
[20]	✓	✗	✗
[21]	✓	✗	✗
[22]	✓	✗	✗
This work	✓	✓	✓

659 As shown in Table 9, previous studies did not satisfy all the criteria. Specifically, for
660 *criterion 1*, all related research makes use of mathematical optimization models to improve

661 livestock production. Particularly for Criterion 1, Raoui *et al.* [11] proposed a customer-
662 centric mathematical model that considers lead times, and destination times in perishable
663 food distribution. Additionally, Jafar *et al.* [12] proposed an approach that integrates
664 nine socio-environmental objectives and 396 decision variables in a watershed management
665 model of the Diyala River basin in Iraq for agriculture and livestock. Also, Chikumbo *et*
666 *al.* [13] addressed the problem of land use optimization for a large agricultural farm, taking
667 into account 14 objectives, including economic, environmental, and social aspects. On the
668 other hand, White *et al.* [14] developed a model that optimizes pasture and nutrition
669 management to examine the environmental impact of beef production. Similarly, Raizada
670 *et al.* [15] used multiple objectives to develop alternative land use plans to maximize farm
671 income, employment, and nutritional security, and minimize soil loss. Also, Zhai *et al.* [17]
672 proposed a model of mission planning considering multiple criteria, such as expected profit,
673 energy consumption and equipment loss, and developed an algorithm called MP-PSOGA,
674 which combines Genetic Algorithms and Particle Swarm Optimization.

675 In addition, Michalak *et al.* [19] used a multi-objective optimization of neural models for
676 vaccine allocation in disease spread scenarios. Also, Li *et al.* [18] defined a multi-objective
677 approach that considers energy and material flows, and addresses economic trade-offs, ef-
678 ficient energy use, and environmental benefits. Furthermore, Chen *et al.* [20] proposed a
679 method that integrates an energy analysis with NSGA-II, evaluating the economic and envi-
680 ronmental trade-offs for sustainable development. Besides, Castonguay *et al.* [21] employed
681 advanced spatial optimization techniques to evaluate the trade-offs between minimizing
682 production costs and reducing greenhouse gas emissions in beef production. Addition-
683 ally, Shahin *et al.* [22] used multi-objective optimization and IoT data mining to quantify
684 greenhouse gas emissions on a dairy farm. Finally, our work focuses on the use of beef
685 production variables to make optimal grazing decisions while maintaining animal welfare.

686 For *criterion 2*, our work is the only one that addresses the problem of rotational grazing
687 using a multi-objective optimization model. Instead of focusing solely on maximizing
688 productivity, the proposed optimization model takes into account forage quality, which is
689 fundamental to ensure adequate nutrition and optimal weight gain in cattle. By considering
690 forage quality as one of the objectives, the model can help determine the optimal allocation
691 of grazing areas to maximize the supply of high-quality forage, which can have a direct
692 impact on the weight gained by the animals.

693 Finally, regarding *criterion 3*, our proposal is the only one that focuses on maximizing
694 weight gain using quality forage, which is taken by our model to calculate the occupancy
695 time of a herd in a pasture and the welfare of the animals through their liberties. Therefore
696 our proposal is the only one that takes into account animal welfare.

697 Using the above-mentioned criteria simultaneously is relevant because they allow ad-
698 dressing the livestock problem from a holistic perspective, considering both efficiency and
699 productivity as well as animal welfare, exploiting all the advances that have been made
700 in precision farming. This will lead to significant improvements in the sustainability and
701 profitability of livestock operations while ensuring the ethical treatment and welfare of
702 animals.

703 This research distinguishes itself from the existing literature by presenting a novel
704 many-objective optimization model for rotational grazing in cattle farming that uniquely
705 integrates considerations of both forage quality and animal welfare freedoms. Contrary
706 to conventional approaches that often place emphasis on efficiency metrics, this approach
707 employs the NSGA-III algorithm to simultaneously boost animal body mass while safe-
708 guarding their health. A 90-day simulation study showing statistically greater average
709 weight gain compared to traditional methods and an improvement in overall animal wel-
710 fare is what demonstrates the feasibility of our proposal.

711 This work is significant to the beef production industry as it introduces a rotational

712 grazing model based on mathematical optimization. Unlike conventional practices, this
713 strategy provides a holistic approach by considering both productive efficiency and respect
714 for livestock living conditions. By adopting a many-objectives approach, the model seeks
715 a balance, resulting in more sustainable and ethical meat production. The results suggest
716 that this strategy is superior in all scenarios, for example in situations of farm underuti-
717 lization as in cases of paddock saturation, offering a more balanced and beneficial approach
718 for the livestock industry.

719 5 Conclusions and Future Work

720 A new approach to rotational grazing of cattle based on mathematical optimization was
721 achieved and showed superior performance to the traditional rotational grazing approach
722 with respect to animal weight gain. The mathematical model was evaluated via simulation
723 using an experimental design with which its effectiveness was statistically proven. Metrics
724 were proposed that characterize the decisions of allocating lots to paddocks during the
725 rotational grazing process that allow measuring animal welfare based on the fulfillment of
726 their freedoms. According to the literature reviewed, this work is the first in which metrics
727 are proposed to measure animal welfare in the livestock context, specifically in the grazing
728 process.

729 Specifically, we have proposed a new approach composed of indices to measure the
730 animal welfare of cattle and the animal weight gain. Our optimization process consists of
731 evaluating the conditions of each paddock on a daily basis and assigning cattle to paddocks
732 in order to maximize cattle weight gain and animal welfare. **Particularly, the rotational
733 grazing allocation model proposed in this work would be integrated into an ACODAT for
734 the management of the meat production process as one of its tasks, which would receive
735 information from other ACODAT tasks on the quality of the pasture and the status of the
736 cattle, among other information, and its assignment would be the decision of ACODAT.**

737 Thanks to the many-objective approach, the proposed rotational grazing strategy al-
738 lows maximizing livestock weight gain while taking care of animal health by maximizing
739 (or minimizing) the proposed metrics associated with animal freedom. This gives a very
740 important added value to the proposal. With the results, it was possible to verify that the
741 proposed model is a significantly superior rotational grazing strategy to the traditional one
742 in any of the evaluated scenarios. Both for cases in which the farm is underutilized and in
743 cases in which the paddocks are saturated.

744 **Thus, the main contribution was to achieve a many-objective model that, in addition
745 to considering the classic objectives of animal fattening, considered animal welfare. For
746 the latter, it was essential to consider the amount of food and space available, as well as
747 the noise levels and temperature of the environment. This modeling allowed us to mainly
748 consider the following freedoms of animals: the freedom to satisfy hunger and thirst, the
749 freedom to engage in normal behavior, and the absence of fear and distress. Other works
750 should be expanded to consider other freedoms related to the absence of discomfort, pain,
751 injury, and illness.**

752 **The simulation and experimental design helped to understand the dynamics of the
753 rotational grazing system and to identify variable forms of interaction between factors that
754 influence livestock weight gain. For example, the effect of the season (rainy or dry) on
755 grass growth directly influences the availability and quality of forage in pastures, which
756 in turn has a great impact on livestock weight. Likewise, the distance traveled by the
757 animals when moving lots of cattle to new pastures has a negative effect on the weight of
758 the animals. All this could be observed in the designed simulation environment.**

759 One of the limitations of the proposal is that the average weight gain of the animals
760 is reduced due to the concern for animal welfare. The multi-objective approach leads to

761 a lower weight gain than could be achieved in a single-objective approach to weight gain.
762 Additionally, the metrics proposed to measure animal welfare in rotational grazing are
763 approximations to measures of freedom that can be improved considering other variables
764 of the context (e.g., climatic).

765 The efficiency of the rotational grazing strategy based on the proposed optimization
766 model was evaluated through a simulation of livestock systems under Colombian tropical
767 conditions. In this context, our approach showed superior performance to the classical
768 grazing strategy. Because of this, our approach will be useful, scalable, and applicable
769 to farms of any size in similar contexts. However, it cannot be assured that its efficiency
770 would have the same quality in grazing systems with very different management practices
771 or significantly different environmental conditions. For example, in grazing systems where
772 water input is a variable or parameter to be modeled because of its variability, the approach
773 proposed in this work would not be easily applicable. To verify the efficiency of our proposal
774 in grazing systems with significantly different management practices, it is necessary to
775 implement the model in a simulation study with parameters according to the context
776 under study.

777 In future work, the incorporation of this dynamic allocation model into an autonomous
778 cycle of data analysis tasks for monitoring the animal fattening process is natural. The
779 autonomous cycle would allow the automation of the animal fattening/rotation process
780 within the framework of precision livestock farming. Also, this is a dynamic optimization
781 problem, so extensions to the optimization model that consider this aspect should be
782 studied in the future. Finally, our many-objective approach allows adding aspects linked
783 to the sustainability of the production process in future work, incorporating environmental
784 factors as objectives to be achieved, such as the reduction of CO_2 emissions, which will
785 improve the applicability and relevance of the proposed approach.

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