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Addendum to Adaptive Codebook Optimization for Beam Training on Off-The-Shelf IEEE 802.11ad Devices

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1 INTRODUCTION

This technical report is an extension to the paper Adaptive Codebook Optimization for Beam Training on Off-The-Shelf IEEE 802.11ad Devices [1]. It provides additional information and more detailed steps for the mathematical derivations in the paper.

2 FORMULATION EXTENSION

We will now explain in more detail the steps to derive full or partial channel state information from the SNR measurements performed with different beam patterns, as explained in Section 4 in [1].

The notation and formulation used in the paper is explained below. We assume a geometric channel for communication denoted by $\mathbf{H}$. Since the reception beam-pattern $\mathbf{c}$ is set to omni-directional during the beam-training, we will develop the formulation for the reception channel defined as $\mathbf{h}_{\text{RX}} = \mathbf{c}^H\mathbf{H}$. We define the beam-pattern for the antenna array, that is, we set the antenna weights $\mathbf{p}'$ chosen from the possible values of the antenna weights of the device. We then normalize signal to meet power constrains $\mathbf{p} = \frac{\mathbf{p}'}{\|\mathbf{p}'\|}$. The values of $\mathbf{p}'$ control the amplifiers and phase shifters of the device (including switching the antennas on and off). The power measured at the receiver side when using the normalized transmission beam-pattern $\mathbf{p}$ is given by $|\mathbf{h}_{\text{RX}}\mathbf{p}|^2$.

2.1 Derivation of 4.1.1 in [1]

We want to prove that if we consider $[\mathbf{a}]_m$ as the power measured by the beam-pattern $\mathbf{p}_m$ defined as

$$[\mathbf{p}_m]_i = \begin{cases} 1 & \text{if } i = \bar{k} \\ e^{(m-1)\frac{\pi}{2}i} & \text{if } i = k \\ 0 & \text{otherwise} \end{cases},$$

where $\bar{k}$ is the index of the reference antenna and k is the antenna element to be measured, and considering as phase reference $\arg[\mathbf{h}_{\text{RX}}]_{\bar{k}} = 0$, we have $\arg([\mathbf{h}_{\text{RX}}]_k) = \arg([\mathbf{f}]_2)$, where $\mathbf{f}$ is the discrete Fourier transform of $\mathbf{a}$, $\mathbf{f} = \mathcal{F}(\mathbf{a})$.

We start from $[\mathbf{a}]_m = |\mathbf{h}_{\text{RX}}\mathbf{p}_m|^2$. Taking into account the expression of $\mathbf{p}_m$, we can develop the scalar product to obtain

$$[\mathbf{a}]_m = |\frac{1}{\sqrt{2}}([\mathbf{h}_{\text{RX}}]_{\bar{k}} + [\mathbf{h}_{\text{RX}}]_k e^{(m-1)\frac{\pi}{2}i})|^2.$$

Taking $[\mathbf{h}_{\text{RX}}]_k$ in its polar form $|[\mathbf{h}_{\text{RX}}]_k|e^{\arg([\mathbf{h}_{\text{RX}}]_k)i}$, we have that

$$[\mathbf{a}]_m = \frac{1}{2}|[\mathbf{h}_{\text{RX}}]_{\bar{k}} + |[\mathbf{h}_{\text{RX}}]_k|e^{(\arg[\mathbf{h}_{\text{RX}}]_k + (m-1)\frac{\pi}{2})i}|^2.$$

We then use the complex expression of the cosine theorem formula $|a+b|^2 = |a|^2 + |b|^2 + 2|a||b|\cos(\arg(b) - \arg(a))$ and the fact that $\arg([\mathbf{h}_{\text{RX}}]_{\bar{k}}) = 0$ by definition to obtain

$$[\mathbf{a}]_m = \Gamma + 2\Delta\cos(\arg[\mathbf{h}_{\text{RX}}]_k + (m-1)\frac{\pi}{2})$$

for

$$\begin{aligned} \Gamma &= \frac{[\mathbf{h}_{\text{RX}}]_{\bar{k}}^2 + |[\mathbf{h}_{\text{RX}}]_k|^2}{2} \\ \Delta &= \frac{[\mathbf{h}_{\text{RX}}]_{\bar{k}}|[\mathbf{h}_{\text{RX}}]_k|}{2} \end{aligned}.$$

Using the Euler cosine expression $\cos(\alpha) = \frac{e^{\alpha i} + e^{-\alpha i}}{2}$ gives

$$[\mathbf{a}]_m = \Gamma + \Delta e^{(\arg[\mathbf{h}_{\text{RX}}]_k + (m-1)\frac{\pi}{2})i} + \Delta e^{-(\arg[\mathbf{h}_{\text{RX}}]_k + (m-1)\frac{\pi}{2})i}.$$

and consequently the expression

$$[\mathbf{a}]_m = \Gamma + \Delta e^{\arg[\mathbf{h}_{\text{RX}}]_k i} e^{(m-1)\frac{\pi}{2}i} + \Delta e^{-\arg[\mathbf{h}_{\text{RX}}]_k i} e^{-(m-1)\frac{\pi}{2}i}.$$

Now 1, $e^{(m-1)\frac{\pi}{2}i}$ and $e^{-(m-1)\frac{\pi}{2}i}$ are the first, second and forth coefficients of a discrete Fourier decomposition, respectively, so we have the expression

$$\mathbf{f} = \mathcal{F}(\mathbf{a}) = [\Gamma, \Delta e^{\arg[\mathbf{h}_{\text{RX}}]_k i}, 0, \Delta e^{-\arg[\mathbf{h}_{\text{RX}}]_k i}]^T.$$

Thus, by measuring $\mathbf{a}$ and computing $\mathbf{f}$ we can extract $\arg([\mathbf{h}_{\text{RX}}]_k) = \arg([\mathbf{f}]_2)$.

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2.2 Derivation of 4.1.2 in [1]

As a reminder, recall that a beam-pattern $\mathbf{p}$ is suitable as a reference if it has higher directionality even with one antenna element k switched off, than the beam pattern resulting from using only that single antenna k. That is, the power contribution of any one of the antenna elements k, $|[\mathbf{h}_{\text{RX}}]_k|$, compared to that of the beam-pattern with that antenna switched off

$$[\mathbf{p}'_k]_i = \begin{cases} 0 & \text{if } i = k \\ [\mathbf{p}']_i & \text{otherwise} \end{cases} \tag{1}$$

is smaller, i.e.,

$$|[\mathbf{h}_{\text{RX}}]_k| \leq |\mathbf{h}_{\text{RX}}\mathbf{p}'_k|, \quad \forall k. \tag{2}$$

Note that switching off the antenna element k does not require it to be active in the beam pattern in the first place. We thus have two cases $|[\mathbf{p}']_k| = 0$ and $|[\mathbf{p}']_k| = 1$.

2.2.1 Formulation for the case $|[\mathbf{p}']_k| = 0$. We want to prove that for a suitable reference beam-pattern $\mathbf{p}$ as defined in Equation 2, if $[\mathbf{a}]_m$ is the power measured by the beam-pattern $\mathbf{p}_m$ defined as

$$[\mathbf{p}'_m]_i = \begin{cases} e^{(m-1)\frac{\pi}{2}i} & \text{if } i = k \\ [\mathbf{p}']_i & \text{otherwise} \end{cases},$$

then considering as phase reference $\arg(\mathbf{h}_{\text{RX}}\mathbf{p}) = 0$, we have $\arg[\mathbf{h}_{\text{RX}}]_k = \arg([\mathbf{f}]_2)$ and

$$|[\mathbf{h}_{\text{RX}}]_k| = \frac{\sqrt{\|\mathbf{p}'\|^2 + 1}}{2}\left(\sqrt{[\mathbf{f}]_1 + 2|[\mathbf{f}]_2|} - \sqrt{[\mathbf{f}]_1 - 2|[\mathbf{f}]_2|}\right)$$

for $\mathbf{f} = \mathcal{F}(\mathbf{a})$.

We start from $[\mathbf{a}]_m = |\mathbf{h}_{\text{RX}}\mathbf{p}_m|^2$. Taking into account the $\mathbf{p}_m$ expression, we develop the scalar product to obtain

$$\begin{aligned} [\mathbf{a}]_m &= |\mathbf{h}_{\text{RX}}\mathbf{p}_m|^2 \\ &= \left|\frac{1}{\sqrt{\|\mathbf{p}'\|^2+1}}\left(\mathbf{h}_{\text{RX}}\mathbf{p}' + [\mathbf{h}_{\text{RX}}]_k e^{(m-1)\frac{\pi}{2}i}\right)\right|^2 \end{aligned}.$$

Taking into account that by definition $\arg(\mathbf{h}_{\text{RX}}\mathbf{p}') = 0$, we can follow the same steps as for 4.1.1 to reach the expression

$$\mathbf{f} = \mathcal{F}(\mathbf{a}) = [\Gamma, \Delta e^{\arg[\mathbf{h}_{\text{RX}}]_k i}, 0, \Delta e^{-\arg[\mathbf{h}_{\text{RX}}]_k i}]^T,$$

for

$$\begin{aligned} \Gamma = \quad [\mathbf{f}]_1 \quad &= \frac{(\mathbf{h}_{RX}\mathbf{p})^2 + |[\mathbf{h}_{\text{RX}}]_k|^2}{\|\mathbf{p}'\|^2+1} \\ \Delta = \quad |[\mathbf{f}_2]| \quad &= \frac{(\mathbf{h}_{RX}\mathbf{p})|[\mathbf{h}_{\text{RX}}]_k|}{\|\mathbf{p}'\|^2+1} \end{aligned}.$$

This allows to extract $\arg([\mathbf{h}_{\text{RX}}]_k) = \arg([\mathbf{f}]_2)$. Given that by squared binomial expression

$$\begin{aligned} (\mathbf{h}_{RX}\mathbf{p} + |[\mathbf{h}_{\text{RX}}]_k|)^2 &= (\Gamma + 2\Delta)(\|\mathbf{p}'\|^2 + 1) \\ (\mathbf{h}_{RX}\mathbf{p} - |[\mathbf{h}_{\text{RX}}]_k|)^2 &= (\Gamma - 2\Delta)(\|\mathbf{p}'\|^2 + 1) \end{aligned},$$

and that by Equation 2 both left sides of the expression are positive, we can derive

$$\begin{aligned} \mathbf{h}_{RX}\mathbf{p} + |[\mathbf{h}_{\text{RX}}]_k| &= \sqrt{(\Gamma + 2\Delta)(\|\mathbf{p}'\|^2 + 1)} \\ \mathbf{h}_{RX}\mathbf{p} - |[\mathbf{h}_{\text{RX}}]_k| &= \sqrt{(\Gamma - 2\Delta)(\|\mathbf{p}'\|^2 + 1)} \end{aligned},$$

and thus

$$|[\mathbf{h}_{\text{RX}}]_k| = \frac{\sqrt{\|\mathbf{p}'\|^2 + 1}}{2}\left(\sqrt{\Gamma + 2\Delta} - \sqrt{\Gamma - 2\Delta}\right).$$

2.2.2 Formulation for the case $|[\mathbf{p}']_k| = 1$. We want to prove that for a beam-pattern $\mathbf{p}$ suitable as reference satisfying Equation 2, if $[\mathbf{a}]_m$ is the power measured by the beam-pattern $\mathbf{p}_m$ defined as

$$[\mathbf{p}'_m]_i = \begin{cases} e^{(m-1)\frac{\pi}{2}i} & \text{if } i = k \\ [\mathbf{p}']_i & \text{otherwise} \end{cases},$$

then considering as phase reference $\arg(\mathbf{h}_{\text{RX}}\mathbf{p}) = 0$, we have

$$\arg([\mathbf{h}_{\text{RX}}]_k) = \arg([\mathbf{f}]_2) - \arg\left(\gamma + |[\mathbf{h}_{\text{RX}}]_k|[\mathbf{p}']_k e^{(\arg[\mathbf{f}]_2)i}\right).$$

for $\mathbf{f} = \mathcal{F}(\mathbf{a})$ and

$$\begin{aligned} \gamma &= \frac{\|\mathbf{p}'\|}{2}\left(\sqrt{[\mathbf{f}]_1 + 2|[\mathbf{f}]_2|} + \sqrt{[\mathbf{f}]_1 - 2|[\mathbf{f}]_2|}\right) \\ |[\mathbf{h}_{\text{RX}}]_k| &= \frac{\|\mathbf{p}'\|}{2}\left(\sqrt{[\mathbf{f}]_1 + 2|[\mathbf{f}]_2|} - \sqrt{[\mathbf{f}]_1 - 2|[\mathbf{f}]_2|}\right) \end{aligned}.$$

We start from $[\mathbf{a}]_m = |\mathbf{h}_{\text{RX}}\mathbf{p}_m|^2$. Using $\mathbf{p}_m$, we can develop the scalar product to obtain

$$\begin{aligned} [\mathbf{a}]_m &= |\mathbf{h}_{\text{RX}}\mathbf{p}_m|^2 \\ &= \left|\frac{1}{\|\mathbf{p}'\|}\left((\mathbf{h}_{\text{RX}}\mathbf{p}' - [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k) + [\mathbf{h}_{\text{RX}}]_k e^{(m-1)\frac{\pi}{2}i}\right)\right|^2. \end{aligned}$$

Defining $\phi = \arg(\mathbf{h}_{\text{RX}}\mathbf{p}' - [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k)$, we can now follow the same steps as for 4.1.1 to reach the expression

$$\mathbf{f} = \mathcal{F}(\mathbf{a}) = [\Gamma, \Delta e^{(\arg[\mathbf{h}_{\text{RX}}]_k - \phi)i}, 0, \Delta e^{-(\arg[\mathbf{h}_{\text{RX}}]_k - \phi)i}]^T,$$

for

$$\begin{aligned} \Gamma = \quad [\mathbf{f}]_1 \quad &= \frac{|\mathbf{h}_{\text{RX}}\mathbf{p}' - [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k|^2 + |[\mathbf{h}_{\text{RX}}]_k|^2}{\|\mathbf{p}'\|^2} \\ \Delta = \quad |[\mathbf{f}_2]| \quad &= \frac{|\mathbf{h}_{\text{RX}}\mathbf{p}' - [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k||[\mathbf{h}_{\text{RX}}]_k|}{\|\mathbf{p}'\|^2} \end{aligned}.$$

Using the same expression development, we have that

$$\begin{aligned} |[\mathbf{h}_{\text{RX}}]_k| &= \frac{\|\mathbf{p}'\|}{2}\left(\sqrt{\Gamma + 2\Delta} - \sqrt{\Gamma - 2\Delta}\right) \\ |\mathbf{h}_{\text{RX}}\mathbf{p}' - [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k| &= \frac{\|\mathbf{p}'\|}{2}\left(\sqrt{\Gamma + 2\Delta} + \sqrt{\Gamma - 2\Delta}\right) \end{aligned}.$$

Given that $\arg(\mathbf{h}_{\text{RX}}\mathbf{p}') = 0$, we obtain

$$\begin{aligned} -\phi &= \arg(\mathbf{h}_{\text{RX}}\mathbf{p}' e^{-\phi i}) \\ &= \arg\left(((\mathbf{h}_{\text{RX}}\mathbf{p}' - [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k) + [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k)\, e^{-\phi i}\right) \end{aligned}.$$

Since by definition $\phi = \arg(\mathbf{h}_{\text{RX}}\mathbf{p}' - [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k)$,

$$-\phi = \arg\left(|\mathbf{h}_{\text{RX}}\mathbf{p}' - [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k| + [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k e^{-\phi i}\right).$$

Decomposing $[\mathbf{h}_{\text{RX}}]_k$ into its polar expression and substituting $|\mathbf{h}_{\text{RX}}\mathbf{p}' - [\mathbf{h}_{\text{RX}}]_k[\mathbf{p}']_k|$ by

$$\gamma = \frac{\|\mathbf{p}'\|}{2}\left(\sqrt{\Gamma + 2\Delta} + \sqrt{\Gamma - 2\Delta}\right)$$

as previously computed results in the expression

$$-\phi = \arg\left(\gamma + |[\mathbf{h}_{\text{RX}}]_k|[\mathbf{p}']_k e^{(\arg([\mathbf{h}_{\text{RX}}]_k) - \phi)i}\right).$$

We now use that $(\arg([\mathbf{h}_{\text{RX}}]_k) - \phi) = \arg([\mathbf{f}]_2)$ to obtain the final expression

$$-\phi = \arg\left(\gamma + |[\mathbf{h}_{\text{RX}}]_k|[\mathbf{p}']_k e^{\arg([\mathbf{f}]_2)i}\right),$$

and thus

$$\arg([\mathbf{h}_{\mathrm{RX}}]_k) = \arg([\mathbf{f}]_2) - \arg\left(\gamma + |[\mathbf{h}_{\mathrm{RX}}]_k|[\mathbf{p}']_k e^{(\arg[\mathbf{f}]_2)i}\right).$$

REFERENCES

[1] Joan Palacios, Daniel Steinmetzer, Adrian Loch, Matthias Hollick, and Joerg Widmer. 2018. Adaptive Codebook Optimization for Beam Training on Off-The-Shelf IEEE 802.11ad Devices. In *Proceedings of the 24th ACM Annual International Conference on Mobile Computing and Networking*.